E. Carpenters, 28. Nov 1853.

RECOMMENDATIONS.

TICKNOR'S PRACTICAL MENSURATION,

For the Academies and Common Schools of the United States.

Easton, January 2, 1849.

WE, the undersigned, Teachers of the Public Schools of Easton, Pa. after a careful examination of Mr. Ticknor's "Treatise on Mensuration," do not hesitate in saying that we are decidedly pleased with its arrangement, and in particular with the plainness with which its rules and examples are written. That a work of this kind has been long needed in our public schools, no teacher will deny, since most other publications of the kind, being abstruse and difficult, are better suited for the college than the public school. But this little work, containing all the information necessary for the carpenter, mason, bricklayer, &c., "and which is so happily adapted to the comprehension of school-boys, is just the thing to teach them what they will want to practice when they become men." We therefore hope it may meet with the cordial reception it deserves, by both scholars and teachers. We shall adopt it in our schools immediately.

B. H. SAXTON,
NEWTON KIRKPATRICK,
D. MULFORD JAMES,
CHAS. S. EMMONS,
HENRY GRIFFITHS,
JAS. JAY OKILL, A. M.
Prin. Pub. School, S. Easton, Pa.

City of Reading, January 1, 1849.

We, the undersigned, having examined Mr. Ticknor's Mensuration, are happy in expressing our entire approbation of the work. The arrangement is very judicious; the questions well selected; the number of questions such as will insure a thorough knowledge of each problem, without subjecting the pupil to drudgery; and the *rationale* so simply stated that none will find the study of his book either tedious or dry.

We cordially recommend the work, and hope its circulation will be as extensive as its merit is undoubted.

STEPHEN ENGLISH, *Teacher of the N. Ward Grammar School.*
JOSEPH MALSBERGER, *Principal N. E. Ward.*
JOHN RYAN, *Principal S. E. Ward P. S.*
B. M. HOAG, *Principal N. Ward Pub. School.*
T. STOCKDALE, *Principal Bingaman St. Pub. School.*
GEO. F. SPAYD, *Principal S. W. Ward Grammar School.*

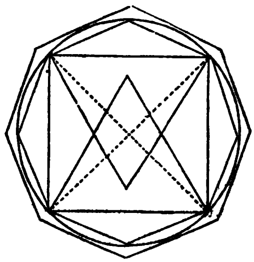
MR. TICKNOR—Dear Sir: I cheerfully endorse the above, and certify further, that your endeavours to bring the practical advantages of science to the mass of the people, in a cheap form, is highly commendable.

GEO. W. F. EMERSON.

TICKNOR'S MENSURATION;

OR,

SQUARE AND TRIANGLE:



BEING A PRACTICAL AND CONCISE SYSTEM

OF

GEOMETRY AND MENSURATION.

ADAPTED TO THE USE OF SCHOOLS AND ACADEMIES
IN THE AMERICAN REPUBLIC.

BY ALMON TICKNOR,

AUTHOR OF THE

COLUMBIAN CALCULATOR, YOUTH'S COLUMBIAN CALCULATOR, COLUMBIAN SPELLING-BOOK, TABLE BOOK, ACCOUNTANT'S ASSISTANT, MATHEMATICAL TABLES, ETC.

"Education is a better safeguard of liberty than a standing army. If we retrench the wages of the schoolmaster, we must raise those of the recruiting sergeant."—HON. EDWARD EVERETT.

POTTSVILLE:
PUBLISHED BY B. BANNAN.
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1849.

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PHILADELPHIA.

REMARKS.

HAVING for many years witnessed the necessity of a *practical elementary* treatise on Geometry and Mensuration in our District Schools and Academies, and as no work answering this description is at present before the public, this, it is presumed, will be a sufficient apology for the attempt to supply a vacancy in this department of science.

Next to a correct knowledge of Arithmetic, we may rank that of Mensuration, a branch of mathematics which embraces a variety of pleasing and interesting subjects of the greatest importance and utility to the man of business, the merchant, artisan, manufacturer, farmer, mechanist, and indeed almost every person whose transactions are of sufficient importance to require the application of figures. Even its introduction into "*Female Seminaries*" should not be considered as derogatory, or "*too rude*" for the exercise of the female mind, as a substitute for less useful studies and employment, when it is known that many ladies have distinguished themselves, as much by their attainments in the abstruse sciences of mathematics and astronomy, as they have by their contributions to our periodical literature. The daily and constant intercourse of men, in the transaction of business, will continually demand the application of the principles of Mensuration, almost as frequently as that of common Arithmetic, and notwithstanding the demand for the application of those rules and principles, it is believed that not more than *one* person in *fifty* is even *partially* acquainted with the science. This *practical* part of mathematics has been quite too much neglected in our district schools as a part of elementary instruction, and seldom receives that attention necessary to acquire a correct knowledge of the science even in many of our "*High Schools and Academies.*" This may be attributed either to the want of *suitable books*, or to the *neglect* of the teacher. In a work of this kind, but *little new* or *original* can be written or expected, beyond the arrangement, the selection of appropriate *practical* questions, and the general adaptation of the volume to the *use* of the man of business, and the *capacity* of those for whom it is intended as an introductory course of instruction to the higher branches of mathematics. In the selection of suitable

matter in the following pages, much care and attention have been bestowed, with a view to introduce such, and such only as appeared the most *useful*, and to omit all *extraneous* subjects as irrelevant in a text-book on elementary science, as well as a fruitful source of complaint with many teachers. A series of examples in *Decimals* and *Evolution* has been introduced as a preliminary course, a correct knowledge of which is *indispensable* in the solution of the following problems, for the reason, that this part of Arithmetic is but *partially*, or *imperfectly* taught by authors or teachers, particularly *decimals*, they being considered of "*no kind of use*," and the time of the pupil is therefore employed on "*mental arithmetic*," and "*foreign currency*," much to the detriment of the pupil, injustice to the public, and a hinderance to the advancement of mathematical science and general information: a system that never has been, nor never can be attended with the *least* benefit to any one, and should be *prohibited* and discountenanced in every school in the Union. This no doubt will be the case when the "public mind" becomes *enlightened* on the *pernicious* consequences, and convinced of the *folly* of a superficial education, and the great loss of *time* and *money* in the vain pursuit of a *phantom*,—"in truth, a most consummate *humbug*."

This volume, with the two "*Columbian Calculators*," embraces about FOUR THOUSAND TWO HUNDRED questions for solution, a large proportion of which are *original*, a number sufficient to make the pupil a *proficient* in the various subjects introduced. If this volume should *merit* and receive the approbation of TEACHERS and others interested, the author will consider himself sufficiently rewarded for all his care and labor, and his most sanguine expectations fully realized.

July 4, 1849.

SIGNS AND TABLES USED IN THIS WORK.

1. Arithmetical Signs.

$+$ *Plus*, or *more*. Sign of addition, as $8 + 4 = 12$; 8 and 4 are 12.

$=$ *Equality*, or equal to. As 3 feet added to 3 feet $=$ 6 feet.

$-$ *Minus*, or *less*. Sign of subtraction; as $8 - 4 = 4$; 4 from 8 and 4 remain.

$\times$ *Into*, or *by*. Sign of multiplication, as $4 \times 2 = 8$; 4 multiplied by 2 is 8.

$\div$ *Divided by*. Sign of division; as $25 \div 5 = 5$; 5 is contained in 25, 5 times, or, $\frac{25}{5} = 5$; $\frac{42}{6} = 7$.

: :: : *Proportion*, as 2 : 4 :: 8 : 16.

$\sqrt{}$ $\sqrt[2]{}$ *Square root*, $\sqrt[2]{16} = 4$, or $16^{\frac{1}{2}} = 4$.

$\sqrt[3]{}$ *Cube root*, $\sqrt[3]{27} = 3$, or $27^{\frac{1}{3}} = 3$.

$\sqrt[4]{}$ *Biquadrate root*, $\sqrt[4]{16} = 2$, or $16^{\frac{1}{4}} = 2$.

2. Land, or Square Measure.

144 inches make - - - - - -	1 sq. foot.
9 feet " - - - - - - - - -	1 sq. yard.
36 feet " - - - - - - - -	1 sq. fathom.
272¼ feet, or 30¼ yards, make - -	1 sq. rod, pole, or perch.
1600 poles " - - - - - - -	1 sq. furlong.
40 poles " - - - - - - -	1 sq. rood.
4 roods " - - - - - -	1 sq. acre.
160 poles, or 4840 yards " - - - - - -	1 sq. acre.
640 acres (1 section) " - - - - - -	1 sq. mile.

3. Distances, or the Chain.

7$\frac{92}{100}$ inches make - - - - - -	1 link.
25 links " - - - - - - - -	1 pole.
100 links " - - - - - - - -	1 chain.
10 chains (of 4 poles*) make - - - - - -	1 furlong.
8 furlengs " - - - - - -	1 mile.
5 rods, or 66 ft., or 100 links, make - - -	1 Gunter's chain.
80 chains, or 320 po. (in length) " - - - -	1 mile.
10 square chains " - - - - -	1 acre.

* Some surveyors use the chains of 2 poles, others of 4 poles.

4. Solid, or Cubical Measure.

By this are measured all things that have *length, breadth,* and *thickness.*

1728 inches make - - - - - - - - 1 foot.
27 feet " - - - - - - - - - - 1 yard.
40 feet of round timber, or 50 feet of hewn timber make - - - - - - - - - - - 1 ton.
42 feet make - - - - - - - - - - 1 ton of shipping.
16 cubic feet make - - - 1 foot of wood, or one cord foot.
128 solid feet, or 8 feet in length, 4 feet in breadth, and 4 ft. in height, or $8 \times 4 \times 4 = 128 =$ - 1 cord of wood.

5. Long Measure.

12 inches make - - - - - - - - 1 foot.
3 feet " - - - - - - - - - - 1 yard.
5½ yards, or 16½ feet make - - - - - 1 rod, pole, or perch.
40 rods " - - - - - - - - 1 furlong.
8 furlongs " - - - - - - - 1 mile.
3 miles " - - - - - - - - 1 league.
60 geographical, or 69½ statute miles - - - 1 degree.
360 degrees - - - - - - a great circle of the earth.

4 inches make - - - - - - - - 1 hand.
9 inches = - - - - - - - - - - 1 span.
18 inches make - - - - - - - - 1 cubit.
6 feet - - - - - - - - - - - - 1 fathom.

6. Other Measures.

282 cubic inches = - - - - 1 gallon, ale measure.
231 " " = - - - - - - - 1 gallon, wine measure.
268⅘ " " = - - - - - - 1 gallon, dry measure.
24¾ cubic feet, or 16½ feet in length, 1½ in breadth, and 1 foot in height = - 1 perch of stone.
$(16{\cdot}5 \div 2 = 8{\cdot}25 + 16{\cdot}5 = 24{\cdot}75.)$
2150·4252 cubic inches = - - - - 1 bush. strick measure.
2553·6299· " " = - - - - - 1 bush. heaped "
The Winchester bushel is 18·5 inches in diameter, and 8 inches deep.

For convenience, the following table may be used.

Solid, or Cubic Measure.

Cubic Inches.	Cubic Feet.	Cubic Yard.	Cubic Poles.	Cubic Furlong.	Cubic Mile.
1728	= 1				
46656	27	= 1			
7762392	4482⅛	166·375	= 1		
496793088000	287496000	10648000	64000	= 1	
2544358061056000	147197952000	5451776000	32768000	512	= 1

INTRODUCTORY COURSE

TO

MENSURATION.

REMARKS.

It is presumed that the pupil is already well acquainted with common arithmetic; for, without a correct knowledge of that important science, he cannot reasonably expect to make much proficiency in the higher branches of mathematics, satisfactory to himself or creditable to his instructor. But, as this may not have been attended to with sufficient care, there can be no objection to a cursory *review* of the most important rules used in the solution of questions in mensuration. As it is foreign to the subject to enter into an *arithmetical demonstration*, the several *rules* will first be given, as a *reference*, and the questions for solution follow promiscuously, as this method will have a tendency to familiarize the pupil in their use and application.

DECIMAL FRACTIONS.

ADDITION.

Rule.—Write the numbers under each other, observing to place tenths under tenths, hundredths under hundredths, &c. Be particular that the *decimal points* stand directly *under* each other, in a perpendicular line, both in the given numbers and in the *sum* or amount. Then perform the operation the same as in addition of integers.

SUBTRACTION.

Rule.—1. Write the numbers the same as integers, observing that the *decimal points* stand directly under each other.

2. Then subtract the same as in whole numbers, and place the decimal point in the remainder under those above.

MULTIPLICATION.

RULE.—1. Write the multiplicand, and under it the multiplier, in the same manner as simple numbers; then multiply without regard to decimal points.

2. When the multiplication is finished, begin at the *right* hand figure of the product, and count off as many figures toward the left as there are decimal places in the multiplier and multiplicand, and *there* place the decimal point.

3. If the number of places in the product be *less* than the decimal places in the multiplier and multiplicand, *prefix* a sufficient number of *ciphers*, to the left of the product, to equal those of the multiplier and multiplicand, and then place the decimal point to the left of the ciphers.

DIVISION.

RULE.—Divide as in whole numbers, and point off as many figures for decimals in the quotient as the decimal places in the dividend exceed those in the divisor. If the quotient does not contain figures enough, supply the deficiency by prefixing ciphers.

To reduce a decimal to a common or vulgar fraction.

RULE.—Erase the decimal point; then write the decimal denominator under the numerator, and it will form a common fraction, which may be treated in the same manner as other common or vulgar fractions.

To reduce a common or vulgar fraction to a decimal.

RULE.—Annex ciphers to the numerator, and divide it by the denominator. Point off as many decimal figures in the quotient as you have annexed ciphers to the numerator.

To reduce several denominations to the decimal of a higher denomination.

RULE.—1. Multiply by as many as it takes of the *next lower* denomination to make one of the higher, adding in the denominations respectively as you multiply, until they are reduced to the lowest denomination in the question, and this is the dividend.

2. Then take *one* of that denomination of which you wish to make it a decimal and reduce it to the *same* denomination with the one above-mentioned, and this last number is the divisor.

3. Divide as in whole numbers, and the quotient is the answer.

To reduce a decimal to its proper value, or compound number to whole numbers of lower denominations.

RULE.—1. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places for a remainder (counting from the *right*) as there are decimal places in the given decimal, and there make the decimal point.

2. Multiply the remainder (that is, the *decimal*) by the next less denomination, and cut off a remainder as before; continue in this way through all the parts of the integer, and the several denominations standing on the *left* of the decimal points is the answer.

REVIEW.

What are fractions? What are decimal fractions? From what do they arise? Why are they called decimals? Ans. Because they *decrease* in a tenfold ratio, as tenths, hundredths, &c. How are decimals expressed? What is always the denominator of a decimal fraction? What is the point placed before a decimal called? Upon what does the value of a decimal depend? What is the difference between *prefixing* and *annexing* ciphers to decimals? How are decimals read? Repeat the process of addition—subtraction—multiplication—division—reduction, &c.

QUESTIONS.

1. Add 12·34565, 7·891, 2·34, 14, ·0011 together.
2. Add ·7509, ·0074, ·69, ·8408, ·6109 together.
3. Add ·7569, ·25, ·654, ·199 together.
4. Add 71·467, 27·94, 16·084, 98·009, 86·5 together.
5. Add 9607·84, 823·79, ·07965, 74·821 together.
6. Add 19·073, 2·3597, 223, ·0197581, 3478·1, 12·358 together.
7. Add 5·3, 11·973, 49, ·9, 1·7314, 34·3 together.
8. From 125·64000 take 95·58756.
9. From 145·00 take 76·84.
10. From 14·674 take 5·91.
11. From 761·8109 take 18·9118.
12. From 171·195 take 125·9176.
13. From 480 take 245·0075.
14. 3·024 × 2·23
15. 25·238 × 12·17.
16. ·007853 × ·035 = ·000274855.
17. ·007 × ·0008.
18. 25·238 × 12·17.
19. ·84179 × ·0385.
20. 4·18000 ÷ ·1812.
21. 186513·239 ÷ 304·81.

22. Divide 7·25406 by ·957. Ans. 7·58.
23. Reduce 20 poles to the decimal of an acre.
24. Reduce 2 roods, 4 poles to the decimal of an acre.
25. Reduce 3 quarters, 2 nails to the decimal of a yard.
26. Reduce 5 furlongs, 16 poles to the decimal of a mile.
27. Reduce 4·5 calendar months to the decimal of a year.
28. Find the value of ·76442 of a pound Troy.
29. What is the value of 875 of a yard?
30. Find the proper quantity of ·089 of a mile.
31. Reduce $\frac{1}{27}$ to a decimal.
32. Reduce $\frac{17}{20}$ to a decimal.
33. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$, and $\frac{7}{38}$ to decimals.
34. Reduce 7 cwt. 3 qrs. 17 lb. 10 oz. 12 drs. to the decimal of a ton.
35. Reduce 8 feet, 6 inches to the decimal of a mile.
36. Reduce 3 roods, $1\frac{1}{4}$ poles to the decimal of an acre.
37. Reduce $5\frac{1}{2}$ yards to the decimal of a mile.
38. ·88 ÷ 88 = ·01 Ans.
39. 88)·88(100 Ans.
40. 7)·8694(

EVOLUTION.

EVOLUTION, or the extraction of roots, is to find such a number as being multiplied into itself a certain number of times will produce that number: if we resolve 36 into two equal factors, namely, 6 and 6, each of these equal factors is called a root of 36, because $6 \times 6 = 36$, and 6 is the square root of 36. And 27, resolved into three equal factors, 3, 3, and 3, each factor is called a root of 27, because $3 \times 3 \times 3 = 27$, and 3 is the 3d or cube root of 27—and the same of other numbers. A square number cannot have more places of figures than *double* the places of the root, and but *one* less. A cube cannot have more figures than *triple* the number of the root, nor but *two* less.

SQUARE ROOT.

RULE.—1. Separate the given number into periods of two figures, beginning with units.

2. Find the root of the period on the left, and place it in the quotient, and its square under said period, which subtract from the number above.

3. Then bring down the next *period*, (two figures,) and place

it on the right of the remainder, as in Division, and this forms a new dividend.

4. Now double this figure, or root, in the quotient, and place it on the left of the new dividend for a divisor.

5. Then consider how often the divisor is contained in the dividend, omitting the right-hand figure, and place the result on the right of the root in the quotient, and then place this figure on the right of the number produced by doubling for a divisor, and multiply as in Division till the root of all the periods is extracted.

For Decimals.—When decimals occur in the given number, they must be pointed both ways from the decimal point, and the root must consist of as many figures, of whole numbers and decimals respectively, as there are periods of integers or decimals in the given number. When a decimal alone is given, *annex* one cipher, if necessary, so that the number of decimal places shall be equal; and the number of decimal places in the *root* will be equal to the number of *periods* in the given decimal.

For Vulgar Fractions.—1. Reduce mixed numbers to improper fractions, and compound fractions to simple ones, and then reduce the fraction to its lowest terms.

2. Extract the square root of the numerator and denominator separately, if they have exact roots; but if they have not, reduce the fraction to a decimal, and then extract the root, as above, &c.

Proof.—Square the root, and add in the remainder.

CUBE ROOT.

Rule.—1. Separate the given number into periods of three figures each, placing a point over units, then over every *third* figure towards the *left* in whole numbers, and over every third figure towards the *right* in decimals.

2. Find the greatest cube in the first period on the left hand; then placing its root on the right of the number for the first figure of the root, subtract its cube from the period, and to the remainder bring down the next period for a dividend.

3. Square the root already found, giving it its true local value; multiply this square by 3, and place the product on the left of the dividend for a divisor; find how many times it is contained in the dividend, and place the result in the root.

4. Multiply the root already found, regarding its local value by this last figure added to it, then multiply this product by 3, and place the result on the left of the dividend under the divisor; under this result write also the square of the last figure placed in the root.

5. Finally, add these results to the divisor; multiply the sum by the last figure placed in the root, and subtract the product from the dividend. To the right of the remainder bring down the next period for a new dividend; find a new divisor, and proceed with the operation as above.

6. When there is a remainder, *periods of ciphers* may be *annexed.*

NOTE.—If the right-hand period of decimals is deficient, this deficiency must be supplied by ciphers. The root must contain as many places of decimals as there are *periods of decimals* in the given number. The cube root of a *vulgar fraction* is found by extracting the root of its numerator and denominator, or reducing the fraction to a decimal. A mixed number should be reduced to an improper fraction.

PROOF.—Multiply the root into itself twice; add the remainder.

QUESTIONS.

1. What is the square root of 54590·25? Ans. 2345.
2. What is the square root of 3271·4007? Ans. 57·19+
3. What is the square root of 96385163? Ans. 9817+
4. What is the square root of 10342656? Ans. 3216.
5. What is the square root of 964·5192360241? Answer. 31·05671.
6. What is the square root of ·0000316969? Ans. ·00563.
7. What is the square root of $\frac{36}{49}$, $\sqrt[2]{\frac{144}{576}}$, $\sqrt[2]{6\frac{1}{4}}$, $\sqrt[2]{52\frac{9}{16}}$.
8. What is the cube root of 259694072? Ans. 638.
9. What is the cube root of 34328125? Ans. 325.
10. What is the cube root of ·37862135? Ans. ·723+
11. What is the cube root of 34965783? Ans. 327.
12. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
13. What is the cube root of $\frac{5}{277}$, or ·018115942?
14. What is the cube root of $\frac{27}{64}$?
15. What is the cube root of $\sqrt[3]{\frac{125}{729}}$?
16. What is the cube root of $\frac{18961}{81}$?

Note.—This part of Arithmetic is more fully explained in the "Columbian Calculator."

DUODECIMALS, OR CROSS-MULTIPLICATION.

This rule is highly valued by artificers and workmen, particularly carpenters and joiners, in measuring and estimating the value of their work. The dimension being taken in feet, inches, and twelfths. A foot is divided into 12 parts, called inches,

each inch into 12 parts called seconds ($''$), each second into 12 parts called thirds ($'''$), and each third into 12 parts called fourths ($''''$), according to the following table:

12 fourths	($''''$)	make	1 third.
12 thirds	($'''$)	"	1 second.
12 seconds	($''$)	"	1 inch.
12 inches or primes		"	1 foot.

Feet	× by feet	give feet.	
Feet	× by inches	give inches.	
Feet	× by seconds	give seconds.	
Inches	× by inches	give seconds.	
Inches	× by seconds	give thirds.	
Seconds	× by seconds	give fourths.	

RULE.—1. Under the multiplicand write the corresponding denominations of the multiplier; that is, place feet under feet, inches under inches, &c.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, write each product under its respective term; observing to carry *one for every* 12, from each lower denomination to its next superior.

3. Multiply in the same manner with the inches; and set the product of each term *one place* further to the right hand; and carry one for every 12 as before.

4. Work in like manner with the seconds, &c., and the sum of the lines will be the product required.

NOTE.—If there be no feet in the multiple, supply their place with a cipher, and in like manner supply the place of any other denomination. Though the feet obtained by the rule are square feet, the inches are not square inches, but the twelfth part of a square foot, thus, $\frac{12}{144}$.

1. 10 ft. 4 in. × 6 ft. 8 in.
2. 29 ft. 7 in. × 4 ft. 9 in.
3. 12 ft. 3 in. 6″ × 5 ft. 6 in.
4. 18 ft. 6 in. 3″ × 7 ft. 9 in.
5. 4 ft. 7 in. 2″ × 4 ft. 2 in. 3″.
6. 11 ft. 6 in. 5″ × 5 ft. 4 in. 2″.
7. 16 ft. 4 in. 4″ × 8 ft. 3 in. 4″.
8. 11 ft. 2 in. 3″ × 6 ft. 2 in. 5″.

PART FIRST.

THE SQUARE  AND TRIANGLE.

Geometry.

Section 1.—Lines and Angles.

DEFINITIONS.

1. A *point*, is that which has position, but not magnitude.

Fig. 1.
Point.

2. A *right line* is the shortest that can be drawn between any two points, and the shortest distance from A to B.

Fig. 2.

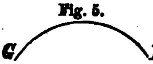

3. *Parallel lines* are at the same distance from each other at every point; they can never meet: C D.

Fig. 3.

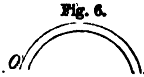

4. A *wave line* is used in the construction of maps, &c.: E F.

Fig. 4.

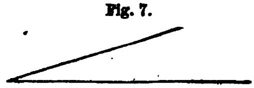

5. A *curve line* is one which changes its direction at every point: G H.

Fig. 5.


6. *Two curve lines* are *parallel*, when they are at the same distance from each other; they will not meet each other: O P.

Fig. 6.


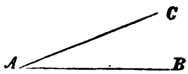
7. *Oblique lines* are those which approach each other, and if prolonged will meet.

Fig. 7.


Horizontal lines are parallel to the horizon, or to the water level.

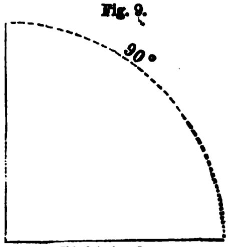
8. *An angle* is the opening, or inclination of two lines which meet each other in a point, as the lines A C, A B, form an angle at the point A. The lines A C and A B are the sides of the angle, and their intersection, A, the *vertex*, or angle A.

Fig. 8.



9. A *perpendicular* is when a straight line meets another straight line, and makes the angles, or both sides of it, equal to each other. The angle formed by the *base* and *perpendicular* is a *right angle*, and contains 90°, or *one-fourth* of a great circle of the earth. $360 \div 4 = 90$. The *base* of a figure is the side on which it stands.

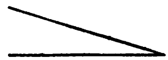
Fig. 9.



Right Angle.

10. An *acute angle* is less than a right angle; the point of intersection is called the *angular point*, which may be greater or less, according as they are more or less inclined or opened.

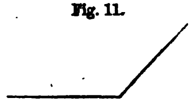
Fig. 10.



Acute Angle.

11. An *obtuse angle* is greater than a right angle.

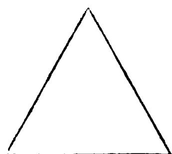
Fig. 11.



Obtuse Angle.

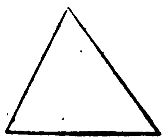
12. An *equilateral triangle* has three sides, all equal.

Fig. 12.



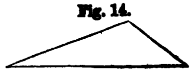
13. An *isosceles triangle* has only two of its sides equal.

Fig. 13.



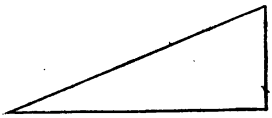
14. A *scalene triangle* has three sides, all unequal.

Fig. 14.



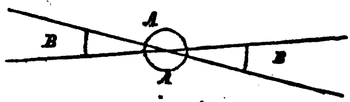
15. A *right-angled triangle* has one right angle; the side opposite, is called the *hypotheneuse*, and the other two sides the *legs*, or the base and perpendicular.

Fig. 15.



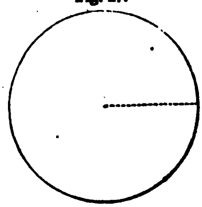
16. *Vertical angles.*—Where two lines intersect each other, the opposite angles A and A are equal to each other, and so also are the angles B and B.

Fig. 16.



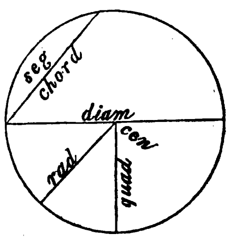
SECTION 2.—CIRCLES AND ANGLES.

Fig. 17.



17. A *circle* is a plain figure, formed by the revolution of a right line about one of its extremities, which remains fixed. It is sometimes called the *circumference.*

Fig. 18.



18. The *radius* of a circle is a straight line drawn from the centre to any part of the circumference.

The *diameter* of a circle is a right line passing through the centre, terminated both ways by the circumference.

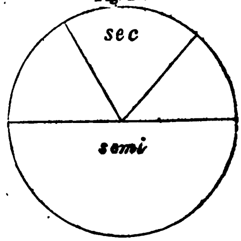
An *arc* of a circle is any part of its *periphery*, or circumference.

A *chord* is a right line joining the extremities of an *arc.*

A *segment* of a circle is any part of a circle bounded by an *arc* and its *chord.*

A *quadrant* is a quarter of a circle.

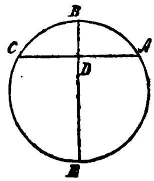
Fig. 19.



19. A *sector* is any part of a circle bounded by an *arc*, and its two radii drawn to its extremities.

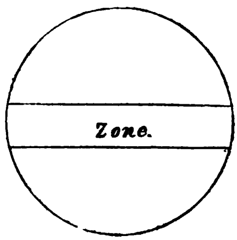
A *semicircle* is one-half of a circle.

Fig. 20.



20. The versed sine or height of an arc, is that part of the diameter contained between the middle of the chord and the arc, as D B=D E.

Fig. 21.

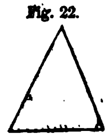


21. A *zone* is a part of a circle included between two parallel chords, and their intercepted arcs.

Section 3.—Plane Figures.

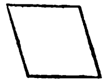
A *plane* figure is a part of a plane, terminated on all sides by lines, either straight or curved. If the bounding lines are straight, the space they enclose is called a *rectilineal* figure, or polygon.

Fig. 22.



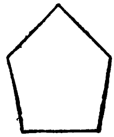
22. A *polygon* of three sides is called a triangle.

Fig. 23.



23. A *polygon* of four sides is called a quadrilateral.

Fig. 24.



24. A *polygon* of five sides is called a pentagon.

A polygon of six sides is called a hexagon.
A polygon of seven sides is called a heptagon.
A polygon of eight sides is called an octagon.
A polygon of nine sides is called a nonagon.
A polygon of ten sides is called a decagon.
A polygon of twelve sides is called a dodecagon.

The lines of a polygon, taken together, are called the *perimeter* of the polygon. The perimeter of a polygon is the sum of all its sides.

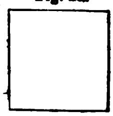
An *equilateral* polygon is one which has all its sides equal.

An *equiangular* polygon is one which has all its angles equal.

All polygons are either *regular* or *irregular* polygons.

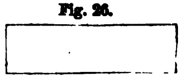
QUADRILATERALS.

Fig. 25.



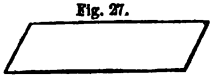
25. A *square* is a quadrilateral, whose sides are all equal, and its angles all right angles. All plain figures bounded by four right lines, are called *quadrangles*, or *quadrilateral*.

Fig. 26.



26. A *rectangle* has its angles right angles, and its opposite sides equal and parallel: called also an "*oblong square.*"

Fig. 27.

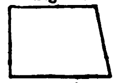


27. A *parallelogram* has its opposite sides parallel, but its angles not right angles.

Fig. 28.

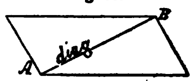
28. A *lozenge* has its sides equal, but is not right angled.

Fig. 29.



29. A *trapezoid*, only two of whose sides are parallel.

Fig. 30.



30. A *diagonal* is a line joining the vertices of two angles, not adjacent; thus, A B is a diagonal, and the angles are equal.

REVIEW.

1. What is a point? 2. A right line? 3. Parallel lines? 4. A wave line? 5. A curve line? 6. Two curve lines? 7. Oblique lines? Horizontal lines? 8. An angle? 9. A perpendicular? The base line? 10. An acute angle? 11. An obtuse angle? 12. An equilateral triangle? 13. An isosceles triangle? 14. A scalene triangle? 15. A right-angled triangle? 16. Vertical angles? 17. A circle? 18. Radius? Diameter? An arc? A chord? A segment? A quadrant? 19. A sector? A semicircle? 20. Versed sine? 21. A zone? A plane figure? 22. A polygon? 23. A polygon of four sides? 24. A polygon of five sides? A polygon of six sides? What of other polygons? What is the perimeter of a polygon? An equilateral polygon? An equiangular polygon? 25. A square? 26. A rectangle? 27. A parallelogram? 28. A lozenge, or diamond? 29. A trapezoid? 30. A diagonal? Circumference? What is altitude? Ans. Height—elevation.

DEFINITION OF TERMS EMPLOYED IN GEOMETRY, &c.

1. An *axiom* is a self-evident proposition.

2. A *theorem* is a truth which becomes evident by means of a train of reasoning, called *demonstration*.

3. A *problem* is a question proposed, which requires *solution*.

4. A *lemma* is a subsidiary truth, employed for the demonstration of a theorem, or the solution of a problem.

5. A *proposition* is applied indifferently to theorems, problems, &c.

6. A *corollary* is an obvious consequence deduced from one or more propositions.

7. A *scholium* is a remark on one or several preceding propositions.

8. A *hypothesis* is a supposition, made either in the enunciation of a proposition, or in the course of a demonstration.

AXIOMS.—1. Things which are equal to the same thing are equal to each other.

2. If equals be added to equals, the whole will be equal.
3. If equals be taken from equals, the remainders will be equal.
4. If equals be added to unequals, the whole will be unequal.
5. If equals be taken from unequals, the remainders will be unequal.
6. Things which are double the same thing are equal to each other.
7. Things which are halves of the same thing are equal to each other.
8. The whole is greater than any of its parts.
9. The whole is equal to the sum of all its parts.
10. All right angles are equal to each other.
11. From one point to another only one straight line can be drawn.
12. Through the same point only one straight line can be drawn, which shall be parallel to a given line.
13. *Magnitudes*, which being applied to each other, coincide throughout their whole extent, are equal.

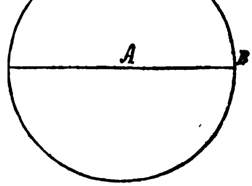
Practical Geometry.

Practical Geometry explains the methods of constructing or describing the geometrical figures by the use of certain instruments.

Section 1.—Problems.

1. *To describe from a given centre the circumference of a circle having a given radius.*

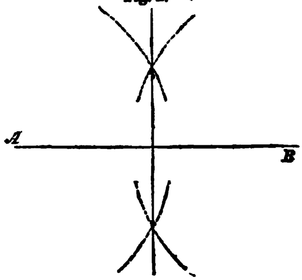
Fig. 1.



Solution.—Let A be the given centre, and A B the given radius. Place one foot of the dividers at A, and extend the other leg until it shall reach to B. Then turn the dividers around the leg at A, and the other leg will describe the required circumference.

2. *To divide a given line A B into two equal parts.*

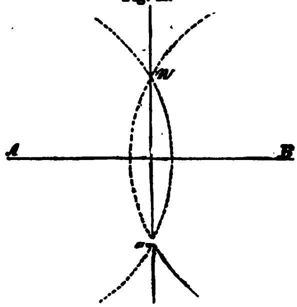
Fig. 2.



Set one foot of the dividers in the point at A, and opening them beyond the middle of the line, describe arches above and below the line; with the same extent of the dividers, set one foot in the point B, and describe two arches crossing the former; draw a line from the intersection of the arches above the line to the intersection below the line, and it will divide the line A B into two equal parts.

Another Method.

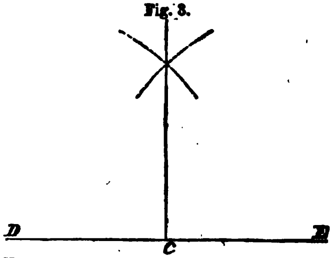
Fig. A.



A. From the points A and B, with any distance greater than half A B, describe *arcs* cutting each other in *n* and *m*, through these points draw the line, &c.

3. *To erect a perpendicular on the point C in a given line.*

Fig. 3.

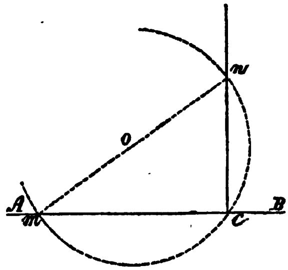


Set one foot of the dividers in the given point C, extend the other foot to any distance at pleasure, as to D, and with that extent make the mark D and E, with the divider's one foot in D, at any extent above the distance of D and E describe an arch above the line, and with the same extent, and one foot in E, describe an arch crossing the former: draw a line from the intersection of the arches to the given point C, which will be perpendicular to the given line in the point C.

4. *When the point is at or near the middle of the line.*

Draw the line A B; then take any point, *o*, and with the distance *o* C, describe the arc *m* C *n*, cutting A B in *m* and C; through the centre, *o*, and the point *m*, draw the line *m o n*, cutting the arc *m* C *n* in *n*; from the point *n*, draw the line *n* C, and it will be the perpendicular required.

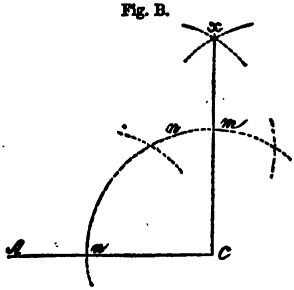
Fig. 4.



Another Method.

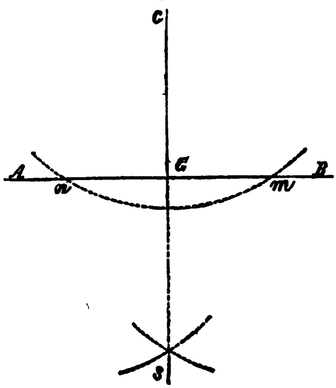
B. From the point C, with any distance more than half the distance of A C, describe the arc *r n m*, cutting the line A C in *r*; with the same distance, and *r* as a centre, cross the arc in *n*, and from *n* in like manner cross it in *m*; from the points *r* and *m* with the same, or any other radius, describe arcs cutting each other in *x*; through the point *x*, draw the line *x* C, and it will be the perpendicular required.

Fig. B.



5. *From a given point C out of a given line A B, to let fall a perpendicular.*

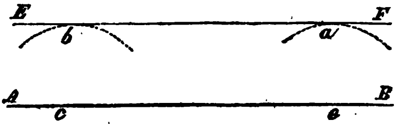
Fig. 5.



From the point C, with any radius, describe the arc *n m*, cutting A B in *n* and *m*; from the points *n m*, with the same or any other radius, describe two arcs cutting each other in *s*; through the point C *s*, draw the line C G *s*, and C G will be the perpendicular required.

6. *To draw a line parallel to a given line.*

Fig. 6.

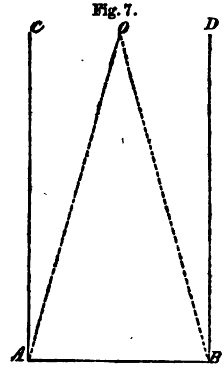


Set one foot of the dividers in any part of the line, as at *c*, extend them at pleasure, unless a distance be assigned, and describe an arc *b*; with the same extent in some other part of the line A B, as at *e*, describe the arc *a*, by a ruler to the extremities of the arcs, and draw the line E F, which will be parallel to the line A B.

NOTE.—To draw parallel lines, use a parallel ruler.

7. *If two straight lines are perpendicular to a third line, they will be parallel to each other, consequently they can never meet.*

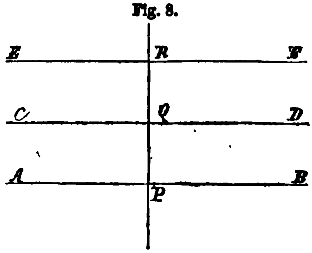
Fig. 7.



Let the two lines A C, B D, be perpendicular to A B, then will they be parallel, for if they could meet in a point O, on either side of A B, there would be two perpendiculars O A, O B, let fall from the same point O, on the same straight line, which is impossible.

8. *Two straight lines which are parallel to a third line, are parallel to each other.*

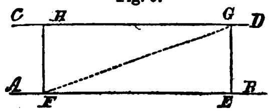
Fig. 8.



Let C D and A B be parallel to the third line E F, then are they parallel to each other. Draw P Q R perpendicular to E F, and cutting A B, C D; since A B is parallel to E F, P R will be perpendicular to A B; and since C D is parallel to E F, P R will, for a like reason, be perpendicular to C D. Hence, A B and C D are perpendicular to the same straight line, consequently they are parallel.

9. *Two parallels are everywhere equally distant.*

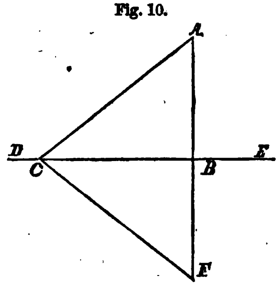
Fig. 9.



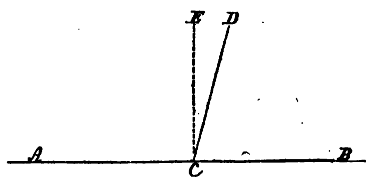
Two parallels A B, C D, being given, if through two points E and F, assumed at pleasure, the straight lines E G, F H be drawn perpendicular to A B, those straight lines will at the same time be perpendicular to C D; if G F be drawn, the angles G F E, F G H, considered in reference to parallels A B, C D, will be alternate angles, and therefore equal to each other.

10. *From a given point without a straight line, only one perpendicular can be drawn to that line.*

Fig. 10.



Let A be the point, and D E the given line; let us suppose that we can draw two perpendiculars, A B, A C, produce either of them, as A B, till B F is equal to A B, and draw F C, then the two triangles C A B, C B F, will be equal, for the angles C B A and C B F are right angles, the side C B is common, and A B equal to B F by construction; therefore the triangles are equal, and the angle A C B is equal to the angle B C F, and the angle A C B is a right angle, therefore B C F must likewise be a right angle. But if the adjacent angles B C A, B C F, are together equal to two right angles, A C F must be a straight line; from whence it follows, that between the same two points A and F, two straight lines can be drawn, which is impossible, (Axiom 11:) neither can two perpendiculars be drawn from the same point to the same straight line.



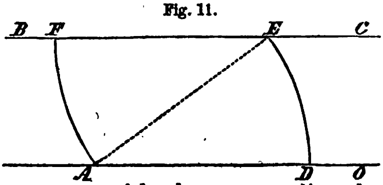
SCHOLIUM.—At a given point C, in the line A B, it is equally impossible to erect two perpendiculars to that line. For if C D, C E, were those two perpendiculars, the angles B C D, D C E, would both be right angles, hence they would both be equal, (Axiom 10;) and the

line C D would coincide with C E; otherwise, a part would be equal to the whole, which is impossible. (Axiom 8.)

NOTE.—Perpendiculars may be raised and let fall by a square, or other suitable instrument.

11. *Through a given point to draw a parallel to a given straight line.*

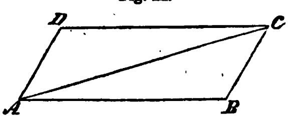
Fig. 11.



Let A be the given point, and B C the given line. From the point A, as a centre, with a radius greater than the shortest distance from A to B C, describe the *indefinite* arc E O; from the point E, as a centre, with the same radius, describe the arc A F; make E D equal to A F, and draw A D, and this will be the parallel required.

12. *In every parallelogram the opposite sides and angles are equal.*

Fig. 12.



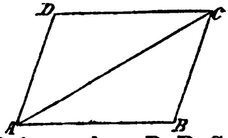
Let A B C D be a parallelogram, then will A B equal D C, A D equal B C, A equal C, and A D C equal A B C. For draw the diagonal B D, the triangles A B D, D B C, have a common side B D; and since A D, B C, are parallel, they have also the angle A D B equal D B C; and since A B, C D, are parallel, the angle A B D is equal B D C; hence the two angles are equal. Therefore the side D C, opposite the angle A D B, is equal to the side D C, opposite the equal angle D B C; and the sides A D, B C, are equal, hence the opposite sides of a parallelogram are equal. Therefore it follows, that the opposite angles of a parallelogram are also equal.

COR. Two parallels A B, C D, included between two other parallels, A D, B C, are equal; and the diagonal D B divides the parallelogram into two equal triangles.

13. *If the opposite sides of a quadrilateral are equal, each to each, the equal sides will be parallel, and the figure will be a parallelogram.*

Let A B C D be a quadrilateral, having its opposite sides respectively equal, namely, A B equal to D C, and A D equal to

Fig. 13.

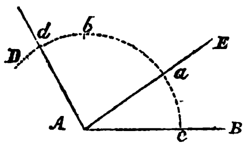


B C, then will their sides be parallel, and the figure a parallelogram; for having drawn the diagonal B D, the triangles A B D, B D C, have all the sides of the one equal to the corresponding sides of the other; therefore they are equal, and the angle A D B, opposite the side A B, is equal to D B C, opposite C D. Also the side A D is parallel to B C; for a like reason A B is parallel to C D, consequently the quadrilateral A B C D is a parallelogram.

Section 2.

14. *To make an angle equal to any number of degrees.*

Fig. 14.

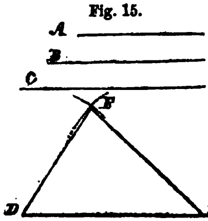


It is required to lay off an acute angle of 35° on a given line A B. Take 60 degrees from the line of chords in the dividers, set one foot in the point A, describe an arch C D at pleasure, then set one foot of the dividers in the brass centre in the beginning of the line of chords, and bring the other to 35° on the line; with this extent set one foot in C; with the other, intersect the arch C D in a, and through a draw the line A E, so will E A B be an angle of 35 degrees. If the angle had been *obtuse*, suppose 125°, then take 90° from the line of chords; set one foot in C, and intersect the arch in C; then take 35° from the same line of chords, and set them from b to d: a line drawn from A through a to E, will make an angle F A B 125 degrees.

Note.—To measure an angle on the line of chords, is only to take the distance on the arch between the lines A B and A E, or A B and A F, and lay it on the line of chords.

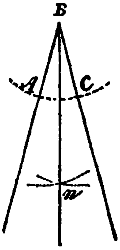
15. *To make a triangle whose sides shall be equal to three given lines.*

Fig. 15.



Let A, B, and C, be the given sides. Draw D E equal to the side A. From the point D as a centre, with a radius equal to the second side B, describe an arc; from E as a centre, with a radius equal to the third side C, describe another arc intersecting the former in F; draw D F and E F, and D E F will be the triangle required.

Fig. 16.

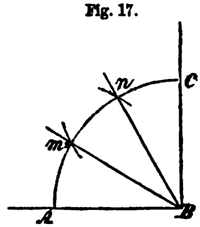


16. *To divide a given angle into two equal parts.*

Let A B C be the given angle; from the point B, with any radius, describe the arc A C, and from A C, with the same, or any other radius, describe arcs cutting each other in *n*; through the point *n* draw the line B *n*, and it will divide the angle A B C as required.

17. *To divide a right angle into three equal parts.*

Fig. 17.

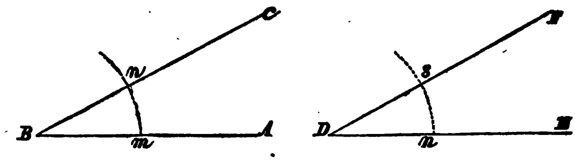


From the point B, with any radius B A, describe the arc A C cutting the legs B A, B C, in A C; and from the point A, with the radius A B, or B C cross the arc A C in *n*; also with the same radius from the point C, cross it in *m*; through the points *m n*, draw the lines B *m*, B *n*, and the angle is divided as required.

18. *At a given point D, to make an angle equal to a given angle A B C.*

From the point B, with any radius, describe the arc *n m* cutting the legs B A, B C, in the points *m n*. Draw the line D E, and from the point D, with the same radius as before, describe

Fig. 18.

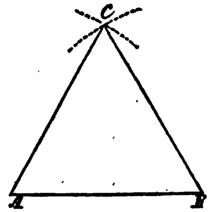


the arc *n s*; take the distance *m n* on the former arc, and apply it to the arc *n s*, from *n* to *s*; through the points D, *s*, draw the line D F, and the angle E D F will be equal to the angle A B C, as was required.

Fig. 19.

19. *Upon a given right line A B, to make an equilateral triangle.*

From the points A and B, with a radius equal to A B, describe arcs cutting in C; draw the lines A C, and the figure A C B will be the triangle required.

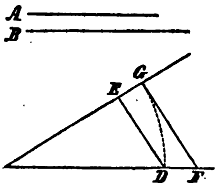


Section 3.

20. *With two given lines A B, to find a third proportional.*

Fig. 20.

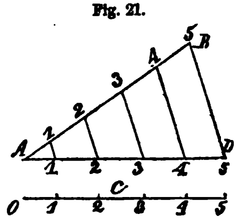
From the point C, draw two right lines, making any angle, C, F G; in these lines take C E, equal to the first term A, and C G, C D, each equal to the second term B: join E D, and draw G F parallel to it; and C F will be the third proportional required.



21. *To divide a given line A B, in the same proportion that another given line C is divided.*

Fig. 21.

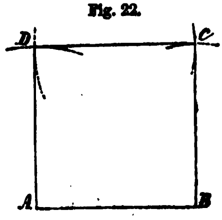
From the point A, draw A D, equal to the given line C, and making any angle with A B to A D, apply the several divisions of C, and join D B, then draw the line 5 5, 4 4, &c.: each parallel to D B and the line A B will be divided as required.



22. *Upon a given line A B, to describe a square.*

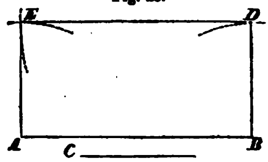
Fig. 22.

From the point B, draw B C perpendicular, and equal to A B: on A and C, with the radius A B, describe two arcs cutting each other in D; draw the lines A D, D C, and the figures A B C D will be the square described.



23. *To describe a rectangle, whose length and breadth shall be equal to two given lines, A B and C.*

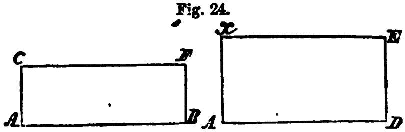
Fig. 23.



At the point B, in the given line A B, erect the perpendicular B D, and make it equal to C: from the point D A, with the radii A B and C, describe two arcs cutting each other in E, then join E A and E D, and A B, D E, will be the rectangle required.

24. *Upon a given line to describe a rectangle that shall be equivalent to a given rectangle.*

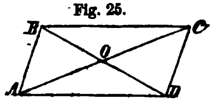
Fig. 24.



Let A D be the line, and A B, F C, the given rectangle. Find a fourth proportional to the three lines, A D, A B, A C, and let A x be that fourth proportional: a rectangle constructed with the lines A D, and A x will be equivalent to the rectangle A B F C.

25. *The two diagonals of a parallelogram bisect each other.*

Fig. 25.



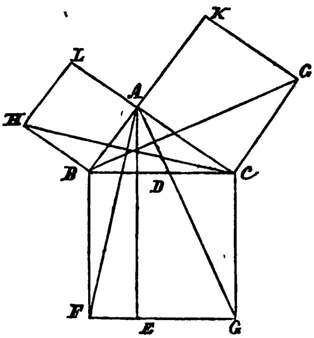
In the parallelogram A B C D, the diagonals A C, B D, bisect each other in the point O, because A B, B D, meet the parallel right lines A D, B C; the angles O A D, O D A, are respectively equal to O C B, O B C, and A D being equal to B C, the triangle O A D, O C B, have the sides O A, O D, respectively equal to O C, O B, and therefore A C, B D, are bisected at the point O.

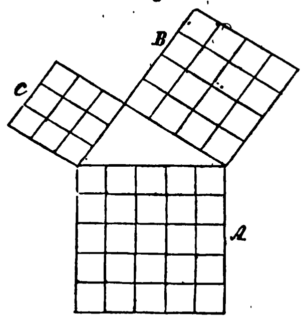
26. *The square described on the hypotenuse of a right-angled triangle is equivalent to the sum of the squares described on the other two sides.* (Fig. B)

Fig. A. Let the triangle A B C be right-angled at A. Having described squares on the three sides, let fall from A, on the hypotenuse, the perpendicular A D, which produce to E; and draw the diagonals A F, C H. The angle A B F is made up of the angle A B C, together with the right angle C B F: the angle C B H is made up of the same angle A B C, together with

the right angle A B H; hence the angle A B F is equal to H B C. But we have A B equal to B H, being sides of the same square B F, equal to B C, for the same reason; therefore the triangles A B F, H B C, have two sides, and the included angle in each equal; consequently they are themselves equal. The triangle A B F is half of the rectangle B E, because they have the same base B F, and the same altitude B D. (Cor. 1.) The triangle H B C is in like manner half of the square A H, for the angles B A C, B A L, being both right angles, A C and A L form one and the same straight line parallel to H B; consequently the triangle H B C, and the square A H, which have

Fig. A. Fig. 26. Fig. B.





B C is equal to A.

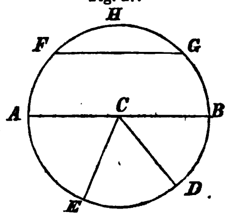
the common base B H, have also the common altitude A B, hence the triangle is half of the square. The triangle A B F has been proved equal to the triangle H B C, hence the rectangle B D E F, which is double of the triangle A B F, must be equivalent to the square A H, which is double of the triangle H B C. In the same manner it may be proved that the rectangle C D E G is equivalent to the square A F. But the two rectangles B D E F, C D E G, taken together, make up the square B C G F, therefore the square B C G F, described on the hypotenuse, is equivalent to the sum of the squares A B H L, A C G K, described in the other two sides; in other words, (Fig. B) the square A contains just as many square feet or yards as are contained in the other two squares B and C.

Section 4.—The Circle and Measurement of Angles.

27. The *circumference of a circle* is a curved line, all the points of which are equally distant from a point within, called

the centre. Every straight line, C A, C E, C D, drawn from the centre to the circumference is called a *radius*, or *semi-diameter*; every line which, like A B, passes through the centre, and is terminated on both sides by the circumference, is called a diameter; it thus follows, that all the radii are equal, and all the diameters are equal also, and each double of the radius.

Fig. 27.

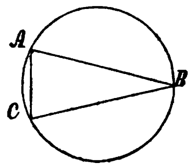


The *arc* is a part of the circumference F H G.

The *chord* of an arc is the straight line F G.

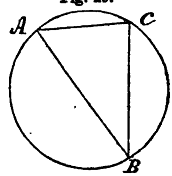
The *segment* is the *surface* or portion of a circle included between an *arc* and its *chord*.

Fig. 28.



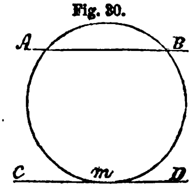
28. An *inscribed triangle* is one which, like B A C, has its three angular points in the circumference.

Fig. 29.



29. An *inscribed angle* is one which has its vertex in the circumference, and is formed by two chords B A C. In general, an *inscribed figure* is one of which all the angles have their vertices in the circumference.

Fig. 30.

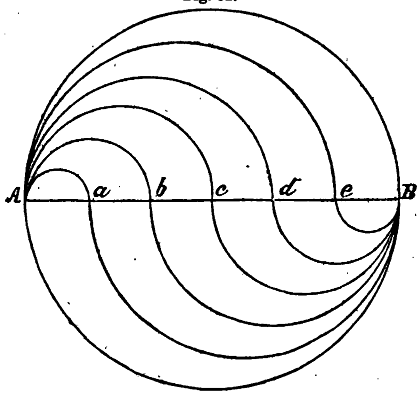


30. A *secant* is a line which meets the circumference in two points, and lies partly within and partly without the circle. A B is a *secant*.

A *tangent* is a line which has but one line in common with the circumference. C D is a tangent. The point *m*, where the tangent touches the circumference, is called the *point of contact*. Two circumferences touching each other have one point in common.

31. *To divide a given circle into any proposed number of parts that shall be equal to each other, both in area and perimeter.*

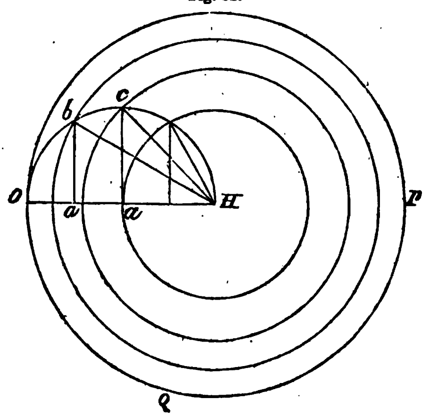
Fig. 31.



Divide the diameter A B into the proposed number of equal parts, at the points *a b c d e*. On A *a*, A *b*, A *c*, A *d*, A *e*, as diameters, describe semicircles on one side of the diameter A B; and on B *e*, B *d*, B *c*, B *b*, B *a*, describe semicircles on the other side of the diameter. Then the corresponding semicircles, joining each other as *above*, will divide the circle in the manner proposed.

32. *To divide a given circle into any number of equal parts by means of concentric circles.*

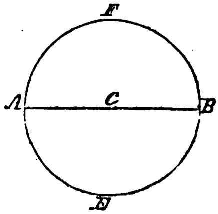
Fig. 32.



Let it be required to divide the circle O P Q into four equal parts. On the radius O H, describe a semicircle; also divide O H into as many equal parts as there may be required; then draw perpendiculars from the points of division to the semicircle on O H; then with the centre H on the radii H *b*, H *c*, &c., describe circles, and the circle is divided as required.

33. *Every diameter divides the circle and its diameter into two equal parts.*

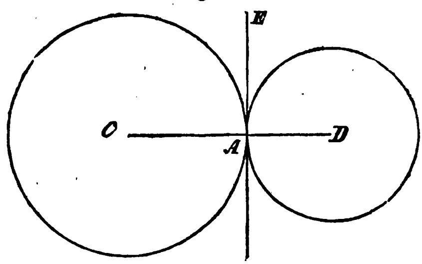
Fig. 33.



Let A E B F be a circle, and A B a diameter: now if the figure A E B be applied to A F B, their common base A B retaining its position, the curve line A E B must fall exactly on the curve line A F B, otherwise there would, in the one or the other, be points unequally distant from the centre, which is contrary to the definition of a circle.

34. *If the distance between the centre of two circles is equal to the sum of their radii, the two circles will touch each other externally.*

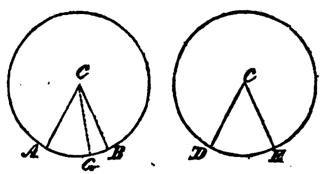
Fig. 34.



Let C and D be the centre, at a distance from each other, equal to C A and A D. The centres will evidently have the point A, common, and they will have no other; because if they had two points common, the distance between their centres must be less than the sum of their radii.

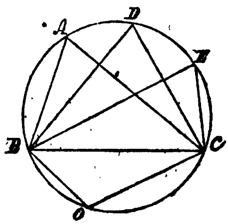
35. *In the same circle, or in equal circles, equal angles having their vertices at the centre intersect equal arcs on the circumference; and conversely, if the arcs intercepted are equal, the angles contained by the radii will also be equal.*

Fig. 35.



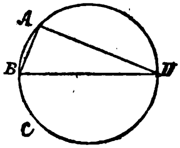
Let C and C be the vertices of equal angles, and the angles A C B equal D C E; since the angles A C B, D C E, are equal, they may be placed upon each other; and since their sides are equal, the point A will evidently fall on D, and the point B on E; but in that case, the arc A B must also fall on the arc D E; for if the arcs did not exactly coincide, there would, in the one or the other, be points unequally distant from the centre; which is impossible; hence the arc A B is equal to D E.

Fig. 36.



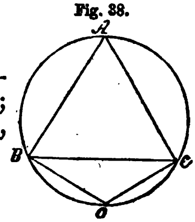
36. Cor. All the angles B A C, B D C, B E C, inscribed in the same segment, are equal; because they are all measured by the half of the same arc B O C.

Fig. 37.



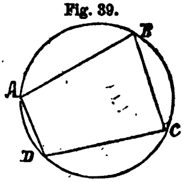
37. Cor. Every angle B A D, inscribed in a semicircle, is a right angle; because it is measured by the half of the semi-circumference B C D, that is, by the fourth part of the whole circumference = 90°, that is one-half of 180°, ¼ of 360°.

Fig. 38.



38. Every angle B O C, inscribed in a segment less than a semicircle, is an obtuse angle; for it is measured by half of the arc B A C, greater than a circumference.

Fig. 39.



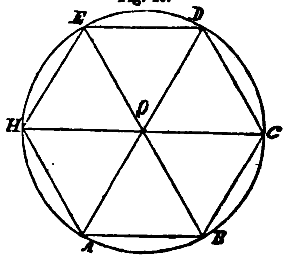
39. Cor. The opposite angles A and C of an inscribed quadrilateral A B C D, are together equal to two right angles; for the angle B A D is measured by half the arc B C D, the angle B C D is measured by half the arc B A D; hence the two angles B A D, B C D, taken together, are measured by the half of the circumference; hence their sum is equal to two right angles.

Section 5.—Polygons.

Def. A *polygon*, which is at once equilateral and equiangular is called a *regular polygon;* regular polygons may have any number of sides; the equilateral *triangle* is one of *three* sides; the *square* is one of *four* sides.

40. *To inscribe a regular polygon of a certain number of sides in a given circle.*

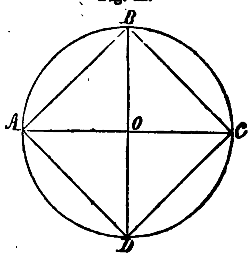
Fig. 40.



Divide the circumference into as many equal parts as the polygon has sides; for the arcs being equal, the chords A B, B C, C D, &c., will also be equal; hence, likewise, the triangles A O B, B O C, C O D, must be equal, because the sides are equal each to each; hence all the angles A B C, B C D, C D E, &c., will be equal; consequently the figure A B C D E, &c. will be a regular polygon.

41. *To inscribe a square in a given circle.*

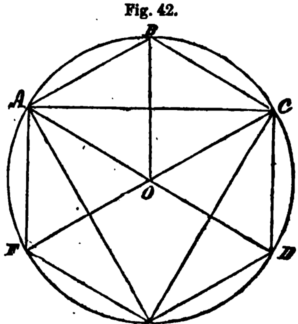
Fig. 41.



Draw two diameters A C, B D, cutting each other at right angles; join their extremities A B C D; the figure A B C D will be a square; for the angles A O B, B O C, &c., being equal, the chords A B, B C, &c., are also equal; and the angles A B C, B C D, &c. being semicircles, are right angles.

42. *In a given circle to inscribe a regular hexagon, and an equilateral triangle.*

Fig. 42.

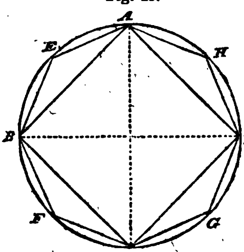


Suppose the problem solved, and that A B is a side of the inscribed hexagon: the radii A O, O B, being drawn, the triangle A O B will be an equilateral. For the angle A O B is the *sixth* part of four right angles: therefore, taking the right angle for unity, we shall have A O B equal $\frac{4}{6}$ equal $\frac{2}{3}$; and the two other angles A B C, B A O, of the same triangle, are together equal $2 - \frac{2}{3}$, or $\frac{4}{3}$; and being mutually equal, each of them must be equal

to $\frac{2}{3}$. Since the triangle A B O is equilateral, therefore the side of the inscribed hexagon is equal to the radius; hence to inscribe a regular hexagon in a given circle, the radius must be applied *six times* to the circumference, which will bring us round to the point of beginning. And the hexagon A B C D E F being inscribed, the equilateral triangle A C E may be formed by joining the vertices of the alternate angles.

43. *To inscribe a square or an octagon in a given circle.*

Fig. 43.

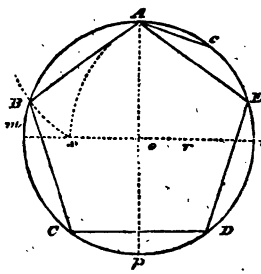


For the Square.—Draw the diameters B D and A C, intersecting each other at right angles; join the points A B, B C, C D and D A, and A B C D will be the square required.

For the Octagon.—Bisect the arc A B of the square in the point E, and the line A E, being carried *eight times* round the circumference, will form the octagon. If the arc A E be again bisected, a polygon may be formed of sixteen sides, and by another bisection a polygon of thirty-two sides, &c.

44. *To inscribe a pentagon or a decagon in a given circle.*

Fig. 44.

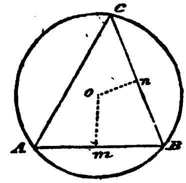


For the Pentagon.—Draw the diameters A *p*, *n m*, at right angles to each other, and bisect the radius *o n* in *r*; from the point *s*, with the distance *r* A, describe the arc A *s*, and from the point A, with the distance A *s*, describe the arc *s* B; join the points A B, and the line A B being carried *five* times round the circle, will form the pentagon required.

For the Decagon.—Bisect the arc A E of the pentagon, in *c*, and the line A *c*, being carried *ten* times round the circumference, will form the decagon required.

45. *About a given triangle A B C, to circumscribe a circle.*

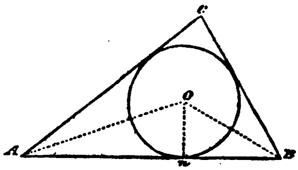
Fig. 45.



Bisect the two sides A B, B C, with the perpendiculars *m o* and *n o*; from the point of intersection *o*, with the distance *o* A, *o* B, or *o* C, describe the circle A C B, and it will be the circle required.

46. *In a given triangle A B C, to inscribe a circle.*

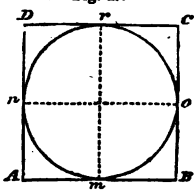
Fig. 46.



Bisect the angles A and B with the lines A O and B O; from the point of intersection O, let fall the perpendicular O *n*, and it will be the radius of the circle required.

47. *To circumscribe a square about a given circle.*

Fig. 47.

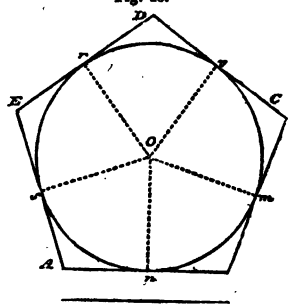


Draw any two diameters *n o* and *r m* at right angles to each other; through the points *m o*, *r n*, draw the lines A B, B C, C D, and D A, perpendicular to *r m* and *n o*, and A B C D will be the square required.

NOTE.—If each of the quadrants be bisected and *tangents* drawn, the circle will be an *octagon*.

48. *About a given circle to circumscribe a pentagon.*

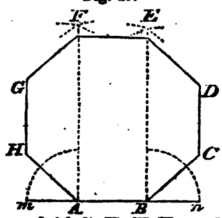
Fig. 48.



Find the points *m n v r s*, (Prob. 43 :) then from the centre *o*, to each of these points, draw the radii *o n*, *o m*, *o v*, *o r*, *o s*. Through the points *n m* draw the lines A B, B C, perpendicular to *o n*, *o m*, producing them till they meet each other at B; in the same manner draw the lines C D, D E, E A, and A B C D E will be the pentagon required.

49. *On a given line A B to form a regular octagon.*

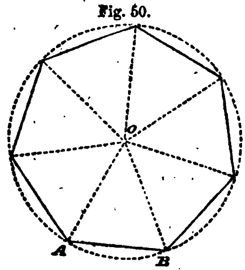
Fig. 49.



On the extremes of the given line A B, erect the indefinite perpendiculars A F and B E; produce A B both ways to m and n, and bisect the angles m A F and n B E, with the lines A H and B C; make A H and B C equal to A B, and draw H G, C D, parallel to A F on B E, and also each equal to A B. From G D, as a centre, with a radius equal to A B, describe arcs crossing A F, B E, in F and E, and if G F, F E, and E D be drawn, A B C D E F G H will be the octagon required.

50. *On a given line A B, to form a regular polygon of any proposed number of sides.*

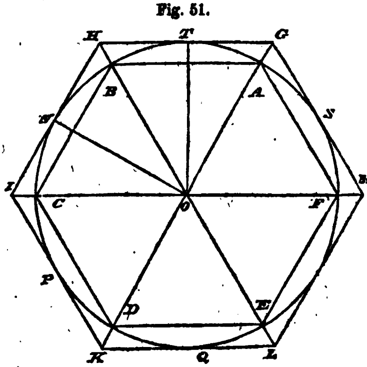
Fig. 50.



Divide 360° by the number of sides, and subtract the quotient from 180°; make the angles A B o and B A o each equal to half the difference last found; from the point of intersection o, with the distance o A or o B, describe the circle; apply the chord A B to the circumference the proposed number of times, and it will form the polygon required.

51. *A regular inscribed polygon being given to circumscribe a similar polygon about the same circle.*

Fig. 51.

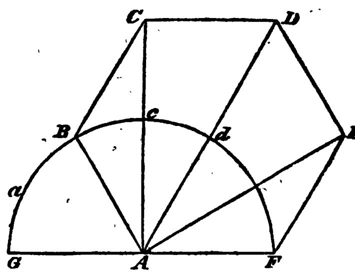


Let C B A F E D be a regular polygon, at T, the middle point of the arc A B, apply the tangent G H, which will be parallel to A B; do the same at the middle point of each of the arcs B C, C D, &c.: these tangents, by their

intersections, will form the regular circumscribed polygon G H, G K, &c., similar to the one inscribed. Since T is the middle point of the arc B T A, and N the middle point of the equal arc B N C, it follows, that B T is equal to B N, or the vertex of the inscribed polygon is at the middle point of the arc N B T. Draw O H; the line O H will pass through the point B.

52. *On a given line A F, to describe a regular polygon of any proposed number of sides.*

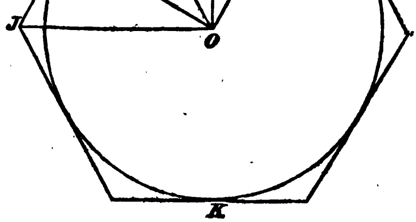
Fig. 52.



From the point A, with the distance A F, describe the semicircle F B G, which will divide into as many equal parts G *a*, *a* B, B *c*, &c. as the polygon is to have sides. From A to the second point of division draw A B, and through the other points *c*, *d*, *e*, &c. draw the lines A C, A D, A E, &c.: apply the distance A F from F to E, from E to D, and from D to C, &c., and join B C, C D, D E, &c., and A B C D, &c. will be the regular polygon required.

53. *The area of a regular polygon is equal to its perimeter multiplied by half of the radius of the inscribed circle.*

Fig. 53.



In the regular polygon G H J K, and O N, O T, radii of the inscribed circle. The triangle G O H will be measured by G H, multiplied by half of O T: the triangle O H J by H J, multiplied by half of O N; but O N is equal to O T, hence the two triangles taken together, will be measured by G H added to H T, multiplied by half of O T. And by continu-

ing the same operation for the other triangles, it will appear that the sum of them all, or the whole polygon, is measured by the sum of the bases G H, H T, &c., or the *perimeter* of the polygon multiplied into one-half of O T, or half the radius of the inscribed circle.

SCHOLIUM. The radius O T, of the inscribed circle, is nothing else than the perpendicular let fall from the centre on one *of the sides; it is sometimes named the apothem of the polygon.*

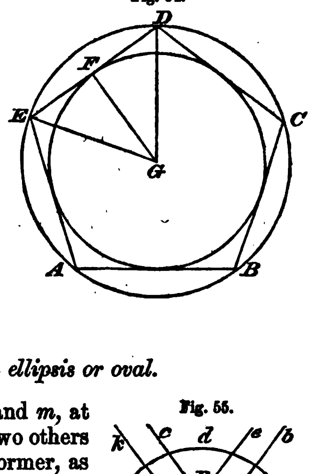
PERIMETER. The sum of the sides, the circumference.

VERTEX. Zenith, point over-head, top.

CONVEX. Rising in a circular form, outside.

CONCAVE. Hollow on the inside.

Fig. 54.

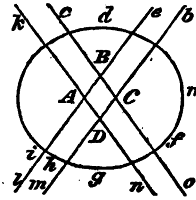


54. *To describe a circle within or without a regular polygon.*

Bisect any two angles A E D, E D C, by the lines E G, D G, and from G let fall G F, perpendicular to the side E D. Then with the radius G E, describe the outer circle, and with the radius G F, describe the inner circle.

55. *To describe an ellipsis or oval.*

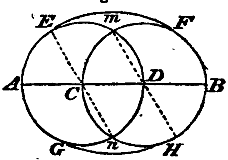
Fig. 55.



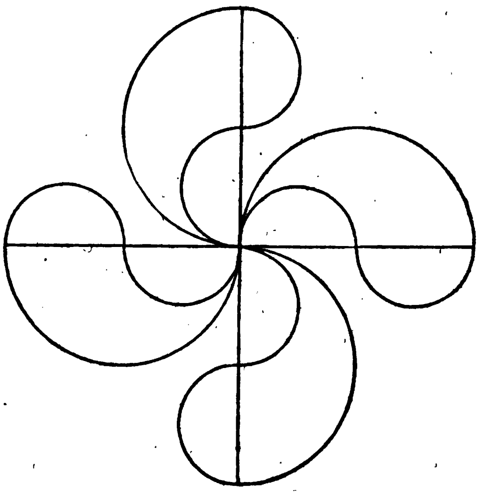
Draw two parallel lines, as *l* and *m*, at a moderate distance; then draw two others at the same distance across the former, as *n* and *o*; by the crossing of their lines will be made a figure A B C D, of four sides; extend the dividers at pleasure, and setting one foot in D describe the arch *c d e*; with the same extent set one foot in B, and describe the arch *f g h*; then set one foot in C, and contract them so as to reach the point *e*, and describe the arch *b m*; with the same extent and one foot in A describe the arch *i k*, and the oval will be completed. In the same manner, with a greater or less extent of chord, may a greater or less oval be made by the same four-sided figure.

Another Method.

Fig. 56.



56. Divide A B into three equal parts, A C, C D, D B; and from the points C, D, with the radii C A, D B, describe the circles A G D E and C H B F. Through the intersections *m*, *n*, and centres C, D, draw the lines *m* H, *n* E; and from the points *n*, *m*, with the radii *n* E, *m* H, describe the arcs E F, H G, and A G H B F E will be the oval required.



Pears.

REMARKS.—In the following review, which should be by a class of from four to ten, there should be a black-board of sufficient length to accommodate all, and each pupil should represent the figure with chalk on the board, and then give the demonstration which he is presumed to have learned previous to recitation.

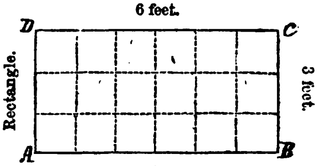
REVIEW.

1. How will you describe the circumference of a given centre and radius? (Draw the figure on the board.) 2. How will you divide a given line into two equal parts? 3. How will you erect a perpendicular on a given point and line? 4. When the point is at or near the middle of the line? 5. How will you let fall a perpendicular from a given

point and given line? 6. How will you draw a line parallel to a given line? 7. If two straight lines are perpendicular to a third line, will they be parallel to each other? 8. If two straight lines are parallel to a third line, will they be parallel to each other? 9. Are two parallels everywhere equally distant? 10. Can more than one perpendicular be drawn from a given point without a straight line? 11. How will you draw a parallel through a given point to a given straight line? 12. In a parallelogram are the opposite sides and angles equal? 13. What is a quadrilateral? A parallelogram? 14. How will you make an angle equal to any number of degrees? 15. How will you make a triangle from three given lines? 16. How will you divide a given angle into two equal parts? 17. How will you divide a right angle into three equal parts? 18. At a given point, how will you make an angle equal to a given angle? 19. On a given right line, how will you make an equilateral triangle? 20. On two given lines, how will you find the third proportional? 21. How will you divide a given line proportionally? 22. How will you describe a square on a given line? 23. How will you describe a rectangle that shall be equal to two given lines? 24. On a given line, how will you describe a rectangle that shall be equivalent to a given rectangle? 25. Will two diagonals in a parallelogram bisect each other? 26. If you describe a square on the hypotenuse of a right-angled triangle, will it be equivalent to the sum of the squares described on the other two sides? 27. What is the circumference of a circle? Radius? Arc? Chord? Segment? 28. What is an inscribed triangle? 29. An inscribed angle? 30. What is a secant? A tangent? 31. How will you divide a given circle into any proposed number of equal parts, &c.? 32. How will you divide a given circle into any number of equal parts by means of concentric circles? 33. Will any diameter divide a circle into two equal parts? 34. When will two circles touch each other externally? 35. What of equal circles and equal angles having their vertices in the centre? 36. Are all the angles equal inscribed in the same segment? 37. What of an angle inscribed in a semicircle? 38. What of an angle inscribed in a segment? 39. What of the opposite angles of an inscribed quadrilateral? What is a polygon? 40. How will you inscribe a regular polygon of a certain number of sides in a given circle? 41. How will you inscribe a square in a given circle? 42. In a given circle, how will you inscribe a regular hexagon, and an equilateral triangle? 43. How will you inscribe a square or an octagon in a given circle? 44. How will you inscribe a pentagon or decagon in a given circle? 45. How will you circumscribe a circle about a given triangle? 46. How will you inscribe a circle in a given triangle? 47. How will you circumscribe a square about a given circle? 48. How will you circumscribe a pentagon about a given circle? 49. How will you form a regular octagon on a given line? 50. How will you form a regular polygon of any proposed number of sides on a given line? 51. A regular inscribed polygon being given, how will you circumscribe a similar polygon about the same circle? 52. On a given line, how will you describe a regular polygon of any proposed number of sides? 53. Is the area of a regular polygon equal to its perimeter? 54. How will you describe a circle either within or without a regular polygon? 55. How will you describe an ellipsis or oval? How by the second method?

Mensuration of Superficies.

SUPERFICIES or surfaces are measured by the superficial inch, foot, yard, &c., according to the measures peculiar to different artists.



$6 \times 3 = 18$ feet.
$(A\ B) \times (B\ C) = A\ B\ C\ D = 18.$

The *area,* or superficial content of any figure, is the space contained within the lines by which the figure is bounded, as in the rectangle A B C D, or by the number of squares contained in it.

The superficial inch, foot, &c., is one inch, one foot, &c. in *length* and *breadth;* and because 12 inches make 1 foot of long measure, therefore $12 \times 12 = 144$ inches make one superficial foot, and $3 \times 3 = 9$ feet, one superficial yard, &c.

PROBLEM 1.—THE SQUARE.

To find the area of a square.

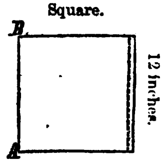
RULE.—Multiply the side by itself; the product will be the area in such terms as correspond with the measure of the sides.

1. How many square feet in a garden 35 feet square?

Thus, $35 \times 35 = 1225$ feet. Answer.

2. How many acres in a piece of land 60 rods square?

$(60 \times 60 = 3600 \div 160 = 22{\cdot}5$ acres.)



$12 \times 12 = 144$ in.
$(A\ B) \times (A\ B.)$

3. Required the area of a square whose side is 5 feet 9 inches.

$$\begin{array}{rl} & 5{\cdot}75 = 9 \text{ in.} \\ & 5{\cdot}75\ \times \\ \hline \text{Feet,} & 33{\cdot}0625 \\ & 12\ \times \\ \hline \text{In.} & 0{\cdot}7500 \\ & 12\ \times \\ \hline \text{Parts,} & 9{\cdot}0000 \end{array}$$

4. What is the area of a square whose side is 35·25 chains?

Thus, 35·25 × 35·25 = 1242·5625 square chains, and 1242·5625 ÷ 10 = 124·25625 acres, or 124 acres, 1 rood, 1 pole. Ans.

5. What is the content of a square field whose side is 46 rods?

Ans. 13 a. 0 p. 36 po.

6. What is the area of a square whose side is 8 ft. 4 in.?

By duodecimals, 69 ft. 5′. 4″. Ans.

7. What is the area of a square field whose side is 50 perches?

Ans. 15 a. 2 r. 20 po.

8. How many men can stand on 9 acres of land, each occupying a space of 3 feet square?

Problem 2.

The area of a square being given, to find the length of the side.

Rule.—Extract the square root of the area.

1. The area of a square is 1728 feet, what is the length of the side?

$\sqrt[2]{1728} = 144$ feet, the side required.

2. How many chains in length is the side of a square containing 125 acres? $\sqrt[2]{125 \cdot 000000} = 35 \cdot 3553 +$ Ans.

3. What is the side of a square field whose area is 7 acres?

Ans. 8·3666 chains.

4. What is the side of a square floor whose area is 1024 feet?

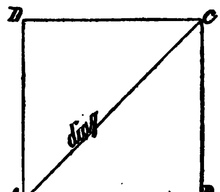
Ans. 32 feet.

5. Required the side of a square field whose area is 12 acres, 2 roods, 16 poles.

Problem 3.

The diagonal of a square being given, to find the area.

Rule.—Square the diagonal and divide by 2, and the quotient will be the area.



1. The diagonal of the square A B C D is 12 chains; required the area.

12 × 12 = 144 ÷ 2 = 72 ÷ 10 = 7 a. 0 r. 32 p. Ans.

2. The diagonal of a square is 16 chains; what is the area? Ans. 12 a. 3 r. 8 po.

3. The diagonal of a square is 12 yards; required the side? Ans. 72.

4. How many acres are contained in a square field whose diagonal is 40 chains? Ans. 80 acres.

5. How many acres are contained in a square field whose diagonal is 55 chains?

Problem 4.

The area of a square being given, to find the diagonal.

Rule.—Extract the square root of double the area.

1. The area of a square piece of land is 84 acres, 2 roods, 16 poles; required the diagonal.

64 a. 2 r. 16 po. = 10336 × 2 = $20372\frac{1}{2}$ = 142 + po. Ans.

2. The area of a square is 128 yards; what is the diagonal?
Ans. 16 yards.

3. The area of a square piece of land is 14 acres, 2 roods, 6 poles; required the diagonal.
Ans. 68·20 + poles.

4. The area of a square meadow is 11 acres; what is the diagonal? Ans. 59·32 poles.

5. The area of a square field of wheat is 52 acres, 3 roods, 16 poles; required the diagonal.

Problem 5.

The diagonal of a square being given, to find the side.

Rule.—Square the diagonal, and extract the square root of half the square.

1. The diagonal of a square is 42 feet; required the side.

Thus, 42 × 42 = 1764 ÷ 2 = 882; $\sqrt{882}$ = 29·69 + feet. Answer.

2. The diagonal of a square is 48 yards; required the side.
Ans. 33·9411 yards.

3. The diagonal of a square is 72 chains; what is the side?
50·9116 chains.

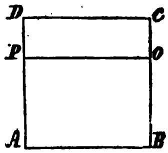
4. What is the side of a square piece of land whose diagonal is 36 perches? Ans. 25·4558 perches.

5. The diagonal of a square is 94 chains; what is the side?

Problem 6.

To cut off a given area from a square, parallel to either side.

Rule.—Divide the given area by the length of the side, the quotient will be the length of the other side to be cut off.



1. What length must be cut from the square A B C D, whose sides are 30 chains, to have an area A B O P of 45 acres at the end?

Thus, 450 chains $\div$ 30 = 15 chains. Ans.

2. The sides of a square are 22 feet; what must be the length of another side to give an area of 132 feet? Ans. 6 feet.

3. The sides of a square are 35 yards; required the length of another side to give an area of 280 yards. Ans. 8 yards.

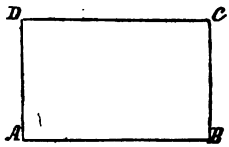
4. The sides of a square are 62 poles; what must be the length of another side to give an area of 2790 square poles? Ans. 45 poles.

5. What length must be cut off from a square field whose sides are 125 perches, to have an area of 50 acres? Ans. 64 perches.

Problem 7.—The Rectangle.

The length and breadth of a rectangle being given, to find the area.

Rule.—Multiply the length by the breadth, and the product will be the area.



1. What is the area of the rectangle A B C D, whose length A B is 16·5 feet, and breadth B C, 12·5 feet?

Thus, 16·5 × 12·5 = 206·25 feet.

2. What is the area of a rectangular board whose length is 112, and breadth 9 inches? Ans. 84 square feet.

3. What is the area of a rectangle whose base is 14 feet 6 inches, and breadth 4 feet 9 inches? Ans. 68 sqr. ft. 10′ 6″, (or by decimals.)

4. How many acres are contained in a rectangular piece of land whose sides are 46 and 58 chains? Ans. 266 a. 3 r. 8 po.

5. The longest side of a rectangular field is 24 rods, and the shortest 16; required the number of acres. Ans. 2 a. 1 r. 24 po.

6. How many square feet in a board 14 feet long, and 15 inches wide? Ans. 17·5 feet.

7. How many acres in a rectangular field whose sides are 36 and 18 chains? Ans. 64 a. 3 r. 8 po.

PROBLEM 8.

The area, and either side of a rectangle being given, to find the other side.

RULE.—Divide the area by the given side, and the quotient will be the other side.

1. The area of a rectangle is 576 feet, and the length 40 feet; required the breadth. $576 \div 40 = 14{\cdot}4$ ft. the breadth. Ans.
2. The area of a rectangle is 846 chains, and its length 42 chains; required the breadth. Ans. $20\frac{1}{7}$ chains.
3. The area of a rectangle is 684 poles, and the length 45 poles; required the breadth. Ans. 15·2 poles.
4. The area of a rectangle is 946 yards, and the shortest side 24 yards; required the longest side. Ans. 39·8 + yards.
5. The area of a rectangle is 1928 feet, and the breadth 42 feet; required the length.

PROBLEM 9.

The area, and the proportion of the two sides of a rectangle being given, to find the sides.

RULE.—Multiply the area by the greater number of the proportion, and divide the product by the less; the square root of the quotient will be the length; then multiply the length by the less number of the proportion, and divide the product by the greater, the quotient will be the breadth.

1. The area of a rectangular piece of land is 432 acres, and the length is to the breadth as 5 to 3; required the sides.

Thus, $432 \times 4 \times 40 = 69120$ perches $\times 5 = 345600 \div 3 = 115200$.

And, $\sqrt[2]{115200} = 339{\cdot}41125$ perches, the length.

Then, $339{\cdot}41125 \times 3 = 1018{\cdot}23375 \div 5 = 203{\cdot}64675$ per. the breadth.

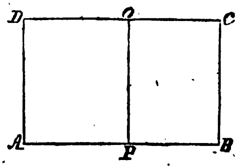
2. The area of a rectangle is 28 acres, and the breadth is to the length as 4 to 7; required the sides.

Ans. 12·6491, and 22·1359 chains.

PROBLEM 10.

The sides of a rectangle being given, to cut off a given area parallel to either side.

RULE.—Divide the area by the side which is to retain its length or breadth, and the quotient will be the length or breadth of the other side.



1. The sides of the rectangle A B C D are 17·14 and 11·22 chains; what must be the length to leave an area B C O P of 12 acres adjoining the breadth?

Thus, 120·000 chains ÷ 11·22 = 10·6951+ chains. Ans.

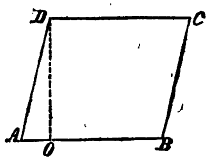
2. The sides of a rectangle are 180 and 75 perches; what must be the breadth so as to leave 22·5 acres adjoining the length? Ans. 20 poles.

3. The sides of a rectangle are 12 and 18 yards; what must be the length to leave 28 square yards adjoining the length?

Problem 11.—The Rhombus.

To find the area of a rhombus.

Rule.—Multiply the length by the perpendicular height, and the product will be the area.



1. The length of a rhombus A B is 12·5 feet, and the perpendicular height D O, 10·75 feet; required the area.

Thus, 10·75 × 12·5 = 134·375 feet.

2. The length of a rhombus is 25½ yds. and the perpendicular height 21½ yards; required the area. Ans. 548·25 yds.

3. What is the area of a rhombus whose length is 19 chains, and height 15 chains? Ans. 28 a. 2 r.

4. What is the area of a rhombus whose base is 12·25 yards, and altitude 8·5 yards? Ans. 104·125 square yards.

5. What is the area of a rhombus whose base is 10·5 chains, and altitude 14·28 chains? Ans. 14 a. 3 r. 30 po.

The Rhomboid. (See preceding rule.*)

1. What is the area of a rhomboid whose length is 7 feet 9 inches, and height 3 feet 6 inches?

Thus, 7·75 × 3·5 = 27·125 square feet.

2. Required the area of a rhomboid whose length is 10·51 chains, and breadth 4·28 chains?

Ans. 44·9028 square chains = 4 a. 1 r. 39·7248 po.

3. How many acres are contained in a rhomboid whose length is 130 perches, and height 57 perches?

Ans. 46 a. 1 r. 10 po.

* The same rule is applied to the square, rectangle, rhombus, rhomboid, and parallelogram, to find the area.

Area of the parallelogram. (Rule as above.)

4. Required the area of a parallelogram whose base is 12 feet 6 inches, and altitude 9 feet 3 inches.

Ans. 115 square feet, 7′ 6″.

5. What is the area of a parallelogram whose base is 8·75 chains, and altitude 6 chains? Ans. 5 a. 1 r. 0 p.

Problem 12.

The area of a rhombus or rhomboid, and the length of the side being given, to find the perpendicular height; or the area and height being given, to find the length of the side.

Rule.—Divide the area by the length of the side, and the quotient will be the perpendicular height; or divide by the height, and the quotient will be the length of the side.

1. The area of a rhombus is 25 perches, and the length of the side 4·5 perches; required the perpendicular height.

Ans. 5·555+ perches.

2. The area of a rhomboid is 5 acres, 1 rood, 20 poles, and the length of the side 35 perches; what is the height?

Thus, 5 a. 1 r. 20 po. = 860 po. ÷ 35 = 24·57+ perches.

3. The area of a rhomboid is 1776 square feet, and the height 24 feet; what is the length? Ans. 74 feet.

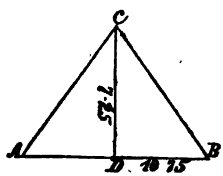
4. The area of a rhombus is 6 acres, and the height 7 chains; required the length of the side.

Problem 13.—The Triangle.

To find the area of a triangle, when the base and perpendicular height are given.

Rule.—Multiply the base by the perpendicular height, and half the product will be the area. Or, which is the same, Multiply the base by half the altitude, and the product will be the area.

1. Required the area of the triangle A B C, whose base A B is 10·75 feet, and altitude C D, 7·25 feet.



Thus, 10·75 × 7·25 = 77·9375 ÷ 2 = 38·96875 area.

Or, 10·75 × 3·625 = half the altitude = 38·96875 area.

2. Required the area of a triangle whose base is 10·5 feet, and height 7 feet 9 inches.

Ans. 40 ft. 8 in. 2 parts.

3. Required the area of a triangle whose base is 12·25 chains, and height 8·5 chains. Ans. 5 a. 0 r. 33 po.

4. What is the area of a triangle whose base is 16·75 feet, and height 6·24 feet? Ans. 52 ft. 3 in. 1 po.

5. Required the number of square yards in a triangle whose base is 40 and altitude 30 feet. Ans. $66\frac{2}{3}$ yards.

6. What is the content of a triangular field whose base is 25·01 chains, and perpendicular 18·14 chains?

Ans. 22 a. 2 r. 29 po.

7. What is the area of a triangular field whose base is $24\frac{1}{2}$ chains, and height 18 chains? Ans. 22 a. 0 r. 8 po.

Note.—The perpendicular height of the triangle is equal to twice the area divided by the base; also, a triangle is half a parallelogram of the same base and altitude; hence the truth of the rule.

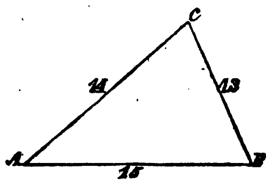
Problem 14.

The three sides of a triangle being given, to find the area.

Rule 1.—Add the three sides together, and take half their sum.

2. From this half sum take each side separately.

3. Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.



1. Required the area of the triangle A B C, whose three sides A B, B C, C A are 15, 13, 14 feet.

Thus, $15 + 13 + 14 = 42 \div 2 = 21$, half the sum of the sides.

Then $21 - 13 = 8$, first difference, or remainder.

$21 - 14 = 7$, second difference, or remainder.

$21 - 15 = 6$, third " " "

Then, $21 \times 8 \times 7 \times 6 = 7056$; and $\sqrt[2]{7056} = 84$ feet, area required.

2. Required the area of a right-angled triangle, whose hypotenuse is 50 yards, and the other two sides 30 and 40 yards.

Ans. 600 yards.

3. The sides of a triangular field are 49 chains, 50·25 chains, and 25·69 chains; required the area.

Ans. 61 a. 1 r. 39·68 po.

4. What is the area of an equilateral triangle whose side is 25 feet? Ans. 270·6329 square feet.

5. What is the area of a triangle whose sides are 22·2, 38, and 40·1 feet? Ans. 413·7114 feet.

Problem 15.

Any two sides of a right-angled triangle being given, to find the third side.

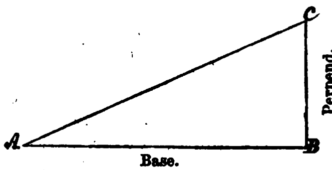
Rule 1.—1. Square each of the sides separately.

2. Add the squares together.

3. Extract the square root of the sum, which will be the *hypotenuse* of the triangle.

Rule 2.—The square root of the difference of the square of the hypotenuse, and either side, will give the other.

Rule 3.—Or multiply the sum of the hypotenuse, and either side, by their difference, and the square root of the product will give the other.



1. The base of a right-angled triangle A B is 50 feet, and the perpendicular B C, 36 feet; required the length of the hypotenuse.

Thus, $50 \times 50 = 2500$; $36 \times 36 = 1296 + 2500 = 3796$. And $\sqrt[2]{3796} = 61{\cdot}6+$ feet. Ans.

2. The hypotenuse of a right-angled triangle is 242 feet, and the perpendicular 182 feet; required the base.

Thus, $242 \times 242 = 58564$; $182 \times 182 = 33124$; $58564 - 33124 = 25440$. And $\sqrt[2]{25440} = 159{\cdot}4989$ feet, base. Ans.

3. The base of a right-angled triangle is 77 yards, and the perpendicular 36 yards; required the hypotenuse.

Ans. 85 yards.

4. The hypotenuse of a right-angled triangle is 109 yards, and the perpendicular 60 yards; required the base.

Ans. 91 yards.

5. The hypotenuse of a right-angled triangle is 25 chains, and the base 20 chains; required the perpendicular.

Ans. 15 chains.

6. The height of a precipice near the side of a river is 108 feet, and a line of 320 feet will reach from the top of it to the opposite bank; required the breadth of the river.

Ans. 302·9703 feet.

7. Required the length of the hypotenuse of a right-angled triangle, if it be 7 feet longer than the perpendicular, when the base is 30 feet.

Thus, $30 \times 30 = 900 + 7 \times 7 = 949 \div 7 \times 2 = 67{\cdot}8$ feet. Answer.

8. The height of a tree standing perpendicularly on a plane is 120 feet; at what height must it break off, so that the top may rest on the ground 40 feet from the base, and the place broken on the upright part.

NOTE.—Square the sum, subtract the square of the base, and divide the remainder by twice the hypotenuse, &c.

Thus, $\overline{120 \times 120 = 14400} - \overline{40 \times 40 = 1600}$; then 12800 $\div (120 \times 2) = 53{\cdot}33$ feet. Ans.

9. The hypotenuse of a right-angled triangle is 315 feet, and the base 289 feet; required the perpendicular.

Ans. 99·1160 feet.

10. Two ships sail from the same port, one due east, 60 miles, the other due north, 80 miles; how far are they apart?

Ans. 100 miles.

PROBLEM 16.

The sum of the hypotenuse and perpendicular, and the base of a right-angled triangle being given, to find the hypotenuse and the perpendicular.

RULE.—To the square of the sum add the square of the base, and divide the number by twice the sum of the hypotenuse and perpendicular, and the quotient will be the hypotenuse. Subtract the hypotenuse from the sum of the hypotenuse and perpendicular, and the remainder will be the perpendicular.

1. A tree 100 feet high, growing perpendicular on a plane, was broken off by a blast of wind; the broken part resting on the upright, and the top on the ground 30 feet from the base; required the length of the upright part.

Thus, $100^2 + 30^2 = 10900 \div (100 \times 2)$ 54·5 feet. Ans. hypotenuse.

$100 - 54{\cdot}5 = 45{\cdot}5$ feet, the perpendicular.

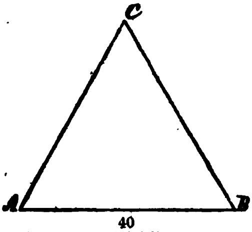
2. The sum of the hypotenuse and perpendicular is 240 yards, and the base 80 yards; required the perpendicular.

Ans. $106\frac{2}{3}$ yards.

PROBLEM 17.

To find the area of an equilateral triangle.

RULE.—Multiply the square of the side by ·433013, and the product will be the area.



1. How many acres in the equilateral triangle whose sides are 40 rods?

Thus, $\cdot433013 \times (40 \times 40 = 1600) = 692\cdot8208$, area in rods $\div 160 = 4\cdot33013$ acres. Ans.

2. Required the area of an equilateral triangle whose side is 25 rods.
Ans. 270·6329 rods.

3. What is the area of an equilateral triangle whose side is 80 perches? Ans. 17·3205 acres.

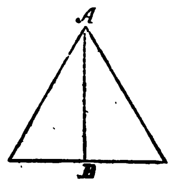
4. How many acres are contained in an equilateral triangle whose side is 16 chains? Ans. 11 a. 0 r. 13·6212 po.

5. What is the area of an equilateral triangle whose side is 25 feet? Ans. 270·6829 square feet.

Problem 18.

The side of an equilateral triangle given, to find the perpendicular.

Rule.—From the square of the given side, subtract the square of half the side, and the square root of the remainder will be the perpendicular.



1. Required the length of a perpendicular, let fall from A to B, when the length of the side is 12 chains.

Thus, $12^2 = 144 - (6^2 = 36)$ half the side. $= 108 : \sqrt{108} = 10\cdot3923 +$ chains. Ans.

The area of an equilateral triangle and the perpendicular given, to find the side.

Rule.—Divide the area by the perpendicular, and the quotient will be the length of the side.

1. The area of an equilateral triangle is 1 acre, 2 roods, 15 poles, and the perpendicular is 21·26 poles; required the length of the side.

Thus, 1 a. 2 r. 15 po. $\times 2 = 510 \div 21\cdot26 = 23\cdot9887 +$ poles. Ans.

The area of an equilateral triangle being given, to find the side.

Rule.—Divide the area by the decimal ·433013, and extract the square root of the quotient.

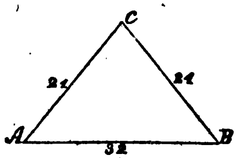
1. The area of an equilateral triangle is 4 acres, 2 roods, 5 poles; how many chains is the length of the side?

Ans. 10·229 chains.

Problem 19.

To find the area of an isosceles triangle, having the length of the side given.

Rule.—From the square of one of the equal sides subtract the square of half the unequal side, and the square root of the remainder will be the length of the perpendicular let fall from the vertical angle to the centre of the base or unequal side; multiply half the length of the base by the perpendicular, and the product will be the area.



1. The two equal sides of an isosceles triangle A B C are 24 feet, and the base 32 feet; how many square feet does the triangle contain?

Thus, $24^2 = 576 : 32 \div 2 = 16$ half the base, $16^2 = 256$ square of the base.

Then $576 - 256 = 320$; and $\sqrt{320} = 17{\cdot}9$ nearly; length of perpendicular.

And $17{\cdot}9 \times 16$ half the base $= 286{\cdot}4$ feet. Ans.

2. Required the area of an isosceles triangle whose base is 20, and each of its equal sides 15. Ans. 111·803.

The area of an isosceles triangle, and the length of the base being given, to find the length of each of the equal sides.

Rule.—Divide twice the area by the base, and the quotient will be the perpendicular; then to the square of the perpendicular add the square of half the unequal side, and the square root of the sum will be the length of each of the equal sides.

1. The area of an isosceles triangle is 48 rods, and the length of the base 8 rods; required the length of each of the equal sides.

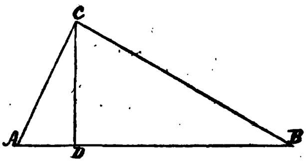
Thus, $48 \times 2 = 96 \div 8 = 12$ per. $12 \times 12 = 144 : 8 \times 8 = 64$ square of the base. $64 \div 2 = 32 + 144 = 176$; and $\sqrt{176} = 13{\cdot}26$ rods. Ans.

2. The area of an isosceles triangle is 5 acres, and the length of the base 20 rods; required the length of the equal sides.

Problem 20.

To find the area of a scalene triangle, the base and perpendicular being given.

Rule.—Multiply the base by half the perpendicular, and the product will be the area.



1. The base of the scalene triangle A B C is 18 rods, and the perpendicular C D is 9 rods; required the number of square rods in the triangle.

Thus, $18 \times 45 = 81$ rods. Ans.

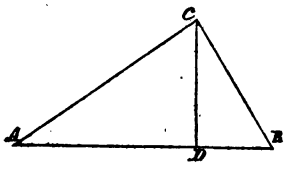
2. The three sides of a scalene triangle are 12, 16, and 20 rods; how many square rods does it contain? Ans. 96 rods.

3. What is the area of a triangle whose base is 20 feet, and height 10·25 feet? Ans. 102·5 feet.

Problem 21.

The area and the base of any triangle being given, to find the perpendicular height. Or the area and height being given, to find the base.

Rule.—Divide twice the area by the base, and the quotient will be the perpendicular height. (Prob. 19, Rule 2.) Or divide by the height, and the quotient will be the base.



1. The area of a triangle A B C is 2 acres, 2 roods, 16 poles, and the base A B, 16 perches; what is the perpendicular height D C?

Thus, 2 a. 2 r. 16 po. $= 416 \times 2 = 832 \div 16 = 52$ perches, the height. Ans.

2. The area of a triangle is 475·5 yards, and the perpendicular 16·5 yards; required the base.

Thus, $475{\cdot}5 \times 2 = 951 \div 16{\cdot}5 = 57{\cdot}63 +$ yards. Ans.

3. The area of a triangle is 5 acres, 0 roods, 33 poles, and the perpendicular 28·5 poles; required the base.

Ans. 58·4561 poles.

4. The area of a triangle is 976·84 poles, and the perpendicular 34 poles; required the base.

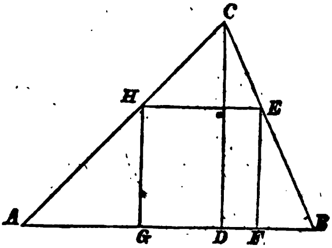
5. The area of a triangular field is 4·7585 acres, and the perpendicular 8·16 chains; required the base.

PROBLEM 22.

The base and perpendicular of any plane triangle being given, to find the side of the inscribed square.

RULE.—Divide the product of the base and perpendicular by their sum, and the quotient will be the side of the inscribed square.

1. The base A B of a triangle is 16 feet, and the perpendicular, D C, 24 feet; what is the side E F of the inscribed square?



Thus, $24 \times 16 = 384$; $16 + 24 = 40$; $384 \div 40 = 9{\cdot}6$ ft. Answer.

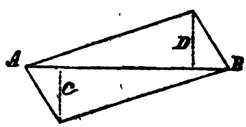
2. The base of a scalene triangle is 30 yards, and the perpendicular 20 yards; required the side of the inscribed square.

PROBLEM 23.—THE TRAPEZIUM.

To find the area of a trapezium.

RULE.—Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and half the product will be the area.

1. Required the area of a trapezium whose diagonal A B is 80·5, and the perpendicular C, 24·5, and D, 30·5.



Thus, $24{\cdot}5 + 30{\cdot}1 = 54{\cdot}6$, the sum of the perpendiculars.

Then $54{\cdot}6 \times 80{\cdot}5 = 4395{\cdot}30 \div 2 = 2197{\cdot}65$. Ans.

2. How many acres in a trapezium whose diagonal is 33 rods, and the sum of the perpendiculars 24 rods?

Ans. 2 a. 1 r. 36 po.

3. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? Ans. 6347 ft. 3 in. = 23·314 sq. feet.

4. Required the area of a trapezium whose diagonal is 33 perches, and the perpendiculars 14 perches, and 23 perches.

Ans. 2 a. 1 r.

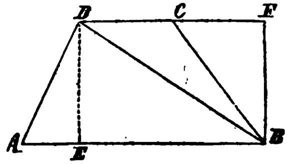
5. Required the area of a trapezium whose diagonal is 45 perches, and the perpendiculars 24 and 32 perches.

6. What is the area in acres of a trapezium whose diagonal is 60 perches, and the perpendiculars 28 and 40·5 perches?

PROBLEM 24.—THE TRAPEZOID.

To find the area of a trapezoid.

RULE.—Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product will be the area.



1. Required the area of the trapezoid A B C D, whose parallel sides D C and A B are 8·22 chains and 12·41 chains, and the perpendicular distance D E, 5·15 chains.

Thus, 12·41 + 8·22 = 20·63 = the sum of the parallel sides.

Then 20·63 × 5·15 = 106·2445 ÷ 2 = 53·12225 sq. chains = 5 a. 1 r. 9·956 po. Ans.

2. Required the area of a trapezoid whose parallel sides are 20·5 and 12·15, and perpendicular distance 10·75.

Ans. 176·03125.

3. The perpendicular distance between the two parallel sides of a trapezoid is 4·5 rods, and the length of the parallel sides are 12·75 rods, and 16·67 rods; required the number of square rods. Ans. 66·195 square rods.

4. Required the area of a trapezoid whose parallel sides are 24·46 chains, and 38·4 chains, and the perpendicular distance 16·2 chains. Ans. 50 a. 3 r. 26 po.

5. Required the area of a trapezoid whose parallel sides are 27·5 chains, and 12·25 chains, and the perpendicular distance 15·40 chains. Ans. 15 a. 0 r. 32·2 po.

6. What is the content when the parallel sides are 20 and 32 chains, and the perpendicular distance 26 chains?

Ans. 67 a. 2 r. 16 po.

7. What is the area of a trapezoid whose parallel sides are 42 chains and 56 chains, and the perpendicular distance 68 chains?

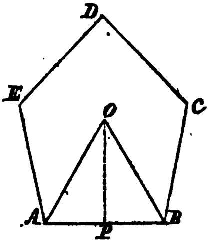
8. Required the area of a quadrilateral in which the diagonal is 42 feet, and the two perpendiculars 18 and 16 feet.

18 + 16 = 34; 42 × 34 = 1428 ÷ 2 = 714. Ans.

PROBLEM 25.—POLYGONS.

To find the area of a regular polygon.

RULE.—Multiply the perimeter, or sum of all the sides of the figure by the perpendicular falling from its centre upon one of the sides, and half the product will be the area.



1. Required the area of the regular pentagon A B C D E, one of whose equal sides A B, B C, &c., is 30 yards, and the perpendicular O P, from its centre, 20·5 yards.

Thus, 30×5 num. sides $= 150 \times 20{\cdot}5$ perches $= 3075{\cdot}0 \div 2 = 1537{\cdot}5$. Ans.

2. Required the area of a regular hexigon (6 sides) whose side is 14·6 feet, and perpendicular 12·64 feet.

Ans. 553·632 feet.

3. Required the area of an octagon whose side is 9·941, and perpendicular 12. Ans. 477·168.

4. How many acres are contained in a regular heptagon whose sides are each 19·38 chains, and perpendicular 20 chains?

Ans. 135·66 acres.

To find the area of a regular polygon when one of its equal sides only is given.

Rule.—Multiply the square of the side of the polygon by the number standing opposite to its name in the following table, and the product will be the area.

No. of sides.	Names.	Areas, or Multipliers.	Radius of inscribed circle.
3	Trigon, or equil. △	0·433013	0·288675
4	Tetragon, or square	1·000000	0·500000
5	Pentagon	1·720477	0·688191
6	Hexagon	2·598076	0·866025
7	Heptagon	3·633912	1·038262
8	Octagon	4·828427	1·207107
9	Nonagon	6·181824	1·373739
10	Decagon	7·694209	1·538842
11	Hendecagon	9·365640	1·702844
12	Duodecagon	11·196152	1·866025

Note.—The multipliers in the table are the areas of the polygons to which they belong, when the side is unity, or 1; hence the square of the side of any polygon multiplied by its tabular number is the area of the polygon.

1. The side of a regular pentagon is 12 feet; what is the area?

Thus, $12 \times 12 = 144$; $1{\cdot}720477 \times 144 = 247{\cdot}748688$ feet, the area.

2. The side of a regular hexagon is 24 feet; what is its area?
Ans. 1496·4917 feet.

3. What is the area of a regular nonagon whose side is 36 inches? Ans. 8011·6439 inches.

4. The side of a pentagon is 25 feet; required the area.
1075·298. Ans.

5. What is the area of a heptagon whose side is 16 feet?
930·28. Ans.

6. The side of a hexagon is 24 feet; required the area.
1496·49. Ans.

7. How many square yards in an octagon whose side is 12 feet 6 inches? Ans. 83·8268+ sq. yards.

8. How many pieces each 4 inches square, may be cut from a decagon whose side is 12 inches? Ans. 69·248 pieces.

9. Each side of a duodecagon is 9 inches; how many square feet does it contain? Ans. 62·98.

10. Required the area of a pentagon whose side is 15?
Ans. 387·107325, area.

11. Required the area of an octagon whose side is 16.
1236·0773. Ans.

12. Required the area of a nonagon whose side is 36.
8011·6439. Ans.

Problem 26.

When the area of any regular polygon is given, to find the side.

Rule.—Divide the area by the number in the table corresponding with the figure, and the square root of the quotient will be the length of the side.

1. The area of a regular pentagon is 4 acres; how many perches are contained in the side?

Thus, 160×4 acres $= 640$ perches; then $640 \div 1{\cdot}720477 = \sqrt{371{\cdot}9898} = 19{\cdot}2870$ perches, the length of the side.

2. Required the length of the side of a hexagon containing 1 acre. Ans. 7·8475 perches.

3. The area of a regular heptagon is 1356·6 yards; what is the side? Ans. 19·3214 yards.

4. The area of a regular octagon is 1642·7 perches; required the length of the side.

Problem 27.—Irregular Figures.

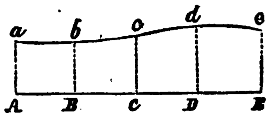
To find the area of a long and irregular figure, bounded on one side by a straight line.

Rule.—Divide the right line or base into any number of equal parts, and measure the breadth of the figure at the points of division, and also at the extremities of the base.

Add together the intermediate breadths, and half the sum of the extreme ones.

Multiply this sum by the base line, and divide the product by the number of equal parts of the base.

1. The breadths of an irregular figure at five equidistant places, A B C D and E, being 8·20 chains, 7·40 chains, 9·20 chains, 10·20 chains, and 8·60 chains, and the whole length 40 chains; required the area.



Thus, $8·20 + 8·60 = 16·80$; $16·80 \div 2 = 8·40$, mean of the extremes; then $8·40 + 7·40 + 9·40 + 10·20 = 35·20$, sum; then $35·20 \times 40$ chains length $= 1408 \div 4 = 352$, square chains.

2. The length of an irregular field is 39 rods, and its breadths, at 5 equidistant places, are 2·4, 2·6, 2·05, 3·65, 3·6 rods respectively; what is the area? Ans. 111·54 rods.

3. The length of an irregular field is 50 yards, and its breadths, at 7 equidistant points, are 5·5, 6·2, 7·3, 6, 7·5, 7, and 8·8 yards; what is the area? Ans. 342·916 sq. yards.

Problem 28.—The Circle.

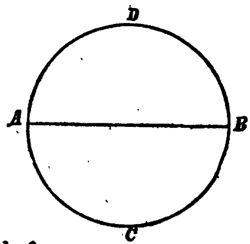
To find the circumference of a circle when the diameter is given, or the diameter when the circumference is given.

Rule.—Multiply the diameter by 3·1416, and the product will be the circumference; or divide the circumference by 3·1416, or multiply the circumference by ·31831, and the result will be the diameter.

Note.—The numbers 3·14159, or 3·1416, ·7854, ·5236, &c., should be made perfectly familiar, in consequence of their frequent use in the solution of superficies and solids in mensuration.

The first expresses the ratio of the *circumference* of a circle to the *diameter*. The second, the ratio of the *area* of a circle to the square of the diameter: and the third, the ratio of the *solidity*

of a sphere to the *cube* of the diameter. Thus, $3{\cdot}1416 \div 4 = {\cdot}7854$; $3{\cdot}11416 \div 6 = {\cdot}5236$.



1. What is the circumference of the circle A B C D, whose diameter, A B, is 7 feet?

Thus, $3{\cdot}1416 \times 7 = 21{\cdot}9912$ ft. the circumference.

2. What is the diameter of a circle whose circumference is 100 yards?

Thus, $100 \div 3{\cdot}1416 = 31{\cdot}831$ yards, the diameter.

Or, $100 \times {\cdot}31831 = 31{\cdot}831$ yards, as before.

3. If the circumference of a circle be 354, what is the diameter? Ans. 112·681.

4. If the diameter of a circle be 17, what is the circumference? Ans. 53·4072.

5. If the circumference of the earth be 25000 miles, what is the diameter? 7958 miles, nearly. Ans.

6. What is the circumference of a wheel whose diameter is 5 feet 2 inches? Ans. 16·2316.

7. What is the diameter of a circle whose circumference is 11652·1944 feet? Ans. 3709 feet.

Problem 29.

To find the area of a circle.

Rule.—Multiply the square of the diameter by ·7854; or, the square of the circumference by ·07958; and the product in either case will be the area.

1. How many square feet are there in a circle whose diameter is 6·5 feet?

Thus, $6{\cdot}5^2 = 42{\cdot}25 \times {\cdot}7854 = 33{\cdot}18315$ square feet. Ans.

2. The circumference of a circle is 11 yards; required the area.

Thus, $11^2 = 132 \times {\cdot}07958 = 10{\cdot}50456$ square yards. Ans.

3. How many square feet are contained in a circle whose diameter is 4 feet 3 inches? Ans. 14·1862875 sq. feet.

4. What is the value of a circular garden whose diameter is 6 rods, at the rate of 8 cents per square foot?

Ans. D. 615·81·6432.

5. The diameter of a circle is 16 chains; how many acres does it contain? Ans. 20 a. 0 r. 16·9984 po.

PROBLEM 30.

The area of a circle being given, to find the diameter or circumference.

RULE.—Divide the area by ·7854, and the square root of the quotient will be the diameter. Or divide the area by ·07958, and the square root of the quotient will be the circumference.

1. The area of a circle is 5 acres, 3 roods, 26 poles; what is the diameter?

Thus, 5 a. 3 r. 26 po. $=$ 946 po. $\div$ ·7854 $= \sqrt{1204{\cdot}4879271}$ $=$ 34·7056 poles, the diameter.

2. The area of a circle being 2 acres, 3 roods, 12 poles; required the circumference.

Thus, 2 a. 3 r. 12 po. $=$ 452 poles.

Then, 452 $\div$ ·07958 $= \sqrt{5679{\cdot}69}$ $=$ 75·3637 poles, the circumference.

3. The area of a circular garden being 1 acre, what is the length of a wall which will enclose it? Ans. 44·839 poles.

4. The area of a circle being 2 acres, 3 roods, 12 poles, what is the circumference? Ans. 75·3657 poles.

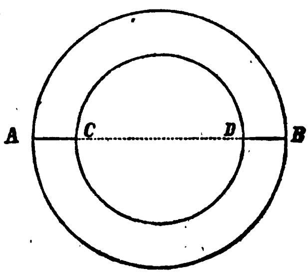
5. The area of a circle being 9 acres, 3 roods, 22 poles, what is the diameter?

PROBLEM 31.

To find the area of a circular ring, or the space included between two concentric circles.

RULE.—Find the areas of the two circles separately. Then the difference of these areas will be the area of the ring. Or multiply the sum of the diameters by their difference, and this product again by ·7854, and it will give the area required.

1. The diameters of the two circles, A B 20 yards, and D C 12 yards; required the area of the ring.



Thus, $20 \times 20 = 400 \times {\cdot}7854$ $= 314{\cdot}16$, area of the outer circle.

Then, $12 \times 12 = 144 \times {\cdot}7854$ $= 113{\cdot}0976$, area of the inner circle.

And, $314{\cdot}16 - 113{\cdot}0976 =$ 201·0624 yards, area of the ring.

2. The diameters of two concentric circles are 8 and 12 yards; what is the area of the ring contained between their circumferences? Ans. 62·832 yards.

3. If the diameters are 20 and 15, what will be the area included between the circumferences? Ans. 137·445.

4. Two diameters are 21·75 and 9·5; required the area of the circular ring. Ans. 300·6609.

5. If the two diameters are 4 and 6, what is the area of the ring? Ans. 15·708.

PROBLEM 32.

The diameter or circumference of a circle being given, to find the side of an equivalent square.

RULE.—Multiply the diameter by ·8862, or the circumference by ·2821; the product in either case will be the side of an equivalent square.

1. The diameter of a circle is 300 yards; what is the side of a square of equal area?

Thus, $300 \times \cdot 8862 = 265\cdot 86$. Ans.

2. The circumference of a circle is 316 yards; what is the side of a square of equal area?

Ans. $316 \times \cdot 2821 = 89\cdot 1436$ yards.

3. A man has a circular meadow, of which the diameter is 875 yards, and wishes to exchange it for a square one of equal size; what must be the side of the square?

Ans. 775·425.

4. The diameter of a circle is 100; what is the side of a square of an equal area? Ans. 88·62.

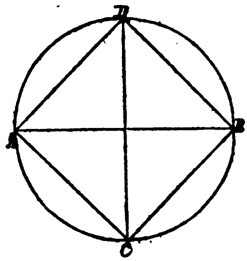
5. The circumference of a circular walk is 64 rods; what is the side of a square containing the same area?

Ans. 18·0544 rods.

PROBLEM 33.

The diameter or circumference of a circle being given, to find the side of the inscribed square.

RULE.—Multiply the diameter by ·7071, or the circumference by ·2251, and the product in either case will be the side of the inscribed square.



1. The diameter A B of a circle, is 525 feet; what is the side A C, or B D of the inscribed square?

Thus, 525 × ·7071 = 371·2275 feet, A C or B D.

2. The diameter of a circle is 239 feet; what is the side of the inscribed square? Ans. 168·9969 feet.

3. The circumference of a circle is 312 feet; what is the side of the inscribed square? Ans. 70·2312 ft.

4. The circumference of a circle is 819 yards; what is the side of the inscribed square? Ans. 184·3569 yards.

5. The diameter of a circle is 65 rods; what is the side of the inscribed square? Ans. 45·9615 rods.

Problem 34.

To find the diameter of a circle equal in area to any given superficies.

Rule.—Divide the area by ·7854, and the square root of the quotient will be the diameter.

1. The length and breadth of a rectangle are 24 and 16 chains; what is the diameter of a circle which contains the same area?

Thus, 24 × 16 = 384, area of the rectangle.

Then 384 ÷ ·7854 = $\sqrt{488·9228}$ = 22·1116 chains, diameter.

2. The three sides of a scalene triangle are 14·18 and 24 yards; what is the diameter of a circle containing the same area? Ans. 12·6267 yards.

3. The length and breadth of a parallelogram are 32 and 18 yards; what is the diameter of a circle that contains the same area? Ans. 27·0810 yards.

Problem 35.

The diameter of a circle being given, to find another containing a proportionate quantity.

Rule.—Multiply the square of the given diameter by the given proportion, and the square root of the product will be the diameter required.

1. The diameter of a circle is 24 chains; what is the diameter of one containing ½ of the area?

Thus, $24 \times 24 = 576 \times \cdot 25 = 144$, and $\sqrt[2]{144} = 12$ chains. Answer.

2. The diameter of a circle is 81 feet; what is the diameter of one containing five times as much? Ans. 181·1215 ft.

3. The diameter of a circle is 9 rods; what is the diameter of one containing 6 times as much? Ans. 22·045 rods.

Problem 36.

To find the length of a circular arc, when the number of degrees and the radius are known.

Rule.—Multiply the number of degrees by the decimal ·01745, and the product arising by the radius of the circle.

1. What is the length of an arc of 30 degrees, in a circle whose radius is 9 feet?

Thus, $\cdot 01745 \times 30 \times \cdot 9 = 4 \cdot 7115$, the length of the arc. Ans.

Note.—When the arc contains degrees and minutes, reduce the minutes to the decimals of a degree, which is done by dividing them by 60.

2. What is the length of an arc of $10° 15'$, or $10\frac{1}{4}°$, in a circle whose diameter is 68? Ans. 6·0813.

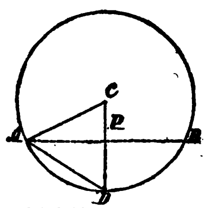
Problem 37.

To find the length of the arc of a circle when the chord and radius are given; or of any arc of a circle.

Rule 1.—Find the chord of half the arc.

2. From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by *three*, and the quotient will be the length of the arc *nearly*.

1. The chord $A\,B = 30$ feet, and the radius $A\,C = 20$ feet; what is the length of the arc $A\,D\,B$?



$A\,C^2 = 20 \times 20 = 400.$

$A\,P^2 = 15 \times 15 = 225$

$\sqrt{775}(13 \cdot 228$ C P.

Then $C\,D - C\,P = 20 - 13 \cdot 228 = 6 \cdot 772 = D\,P$.

And $A\,D = \sqrt{A\,P^2 + P\,D^2} = \sqrt{225 + 45 \cdot 859984}$.

Hence $A\,D = 16 \cdot 4578 =$ the chord of the half arc.

Then $\dfrac{16 \cdot 4578 \times 8 - 30}{3} = 33 \cdot 8874 =$ arc A D B.

2. The chord of an arc is 16, and the diameter of the circle 20; what is the length of the arc? Ans. 18·5178.

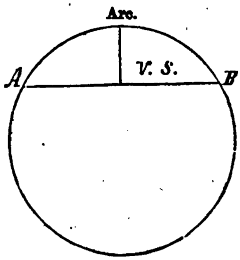
3. Required the length of an arc of 30 degrees, the radius of the circle being 14 feet.

Thus, $30 \times 14 \times \cdot 01745 = 7 \cdot 329$ feet, the length of the arc.

Problem 38.

The chord and versed sine given, to find the diameter of a circle.

Rule.—Divide the square of half the chord by the versed sine, and to the quotient add the versed sine; the sum will be the diameter of the circle.



1. The chord of an arc of a circle is 12 feet (A B), and the versed sine is 2 feet; what is the diameter of the circle?

Thus, $12 \div 2 = 6 \times 6 = 36 \div 2 = 18 + 2$ V. S. $= 20$ feet. Ans.

2. The chord of the arc of a circle is 5 rods, and the versed sine 2 rods; what is the diameter of the circle?

21·53125 rods. Ans.

Problem 39.

The versed sine of an arc, and the diameter of the circle given, to find the chord.

Rule.—From the diameter subtract the versed sine; multiply the remainder by the versed sine; the product will be the square of *half* the chord, the square root of which will give the *half chord.*

1. The diameter of a circle is 30 chains, and the versed sine 3 chains; what is the length of the chord?

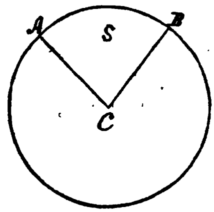
Thus, $30 - 3 = 27 \times 3 = \sqrt[2]{81} = 9 = \frac{1}{2}$ chord, $\times 2 = 18$, chord. Ans.

2. The diameter of a circle is 18 feet, and the versed sine 2 feet; what is the length of the chord? Ans. 11·3136 ft.

Problem 40.

The chord and versed sine given, to find the area of a sector.

Rule.—As 360 degrees are to the circumference of the circle, so are the number of degrees contained in the area of the sector



to the length of the arc; or, as 360 degrees are to the area of the circle, so are the number of degrees contained in the arc of the sector to the area; or, as 360° is to the number of degrees in the arc of the sector, so is the area of the circle to the area of the sector.

1. Required the area of the sector, the chord of half the arc being 30, and the diameter of the circle 100.

Thus, versed sine $= \frac{30^2}{100} = \frac{900}{100} = 9$.

Then $100 \times 60 - 9 \times 27 = 5757$; $\frac{30 \times 2 \times 9 \times 10}{5757} = \frac{5400}{5757}$ $= 9380$, which to twice the chord of half the arc, 60, will make $60{\cdot}9380 =$ the length of the arc $\frac{60{\cdot}9380 \times 50}{2} = \frac{3046{\cdot}9000}{2} =$ $1523{\cdot}45$, the area of the sector. Ans.

Rule 2.—Multiply the number of degrees by the decimal ·01745 and the product arising by the radius of the circle; this will give the length of the arc. Multiply the arc by one-half the radius, and the product will be the area.

2. What is the area of the circular sector A C B, the arc A B containing 18°, and the radius C A being equal to 3 feet?

Thus, $\cdot 01745 \times 18 \times 3 = \cdot 94230 =$ length A B.

Then $\cdot 94230 \times 1{\cdot}5 = 1{\cdot}41345 =$ area.

3. Required the area of a sector whose radius is 25, and the arc one of 147° 29′. Ans. 804·2448.

4. What is the area of a circular sector when the length of the arc is 650 feet, and the radius 325?

105625 square feet. Ans.

5. Required the area of the sector whose height is 4 perches, and the radius of the circle 8 perches.

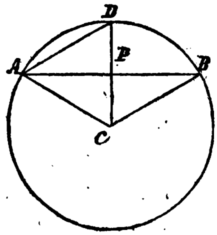
Ans. 66·8581 perches.

Problem 41.

To find the area of a segment of a circle.

Rule 1.—Find the area of the sector having the same arc with the segment, by the last problem.

2. Find the area of the triangle formed by the chord of the segment and the two radii through its extremities.



3. If the segment is greater than the semicircle, add the two areas together; but if it is less, then subtract them, and the result in either case will be the area required.

1. What is the area of the segment A D B, the chord A B = 24 feet, and C A = 20 feet?

Thus, C P = $\sqrt{}$C A = 20^2 = 400 −
A P^2 = 12^2 = 144. −144

$\sqrt{256}$(16 root.

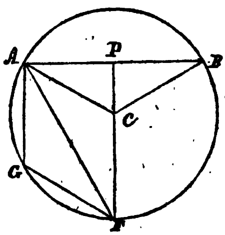
Then P D = C D − C P = 20 − 16 = 4; A D = $\sqrt{}$A P^2 = 144 + P D^2 = 4^2 = 16.

16

$\sqrt{160}$(12·64911, root.

Then arc A D B = $\dfrac{12{\cdot}64911 \times 8 - 24}{3}$ = 25·7309, arc.

Arc A D B =	25·7309 ×	A P = 12 ×
Half radius	10	C P = 16
Area sector A D B C =	257·309	192 = area C A B.
Area =	−192	
	65·309 area of segment A D B.	



2. Find the area of the segment A F B, having the following lines, viz.

A B = 20·5; F P = 17·17; A F = 20; F G = 11·5, and C A = 11·64.

Arc A G F = $\dfrac{\text{F G} \times 8 - \text{A F}}{3}$ = $\dfrac{115 \times 8 - 20}{3}$ = 24.

Sector A G F B C = 24 × 11·64 = 279·36.

But C P = F P − A C = 17·17 − 11·64 = 5·53.

Then area A C B = $\dfrac{\text{A B} \times \text{C P}}{2} = \dfrac{20{\cdot}5 \times 5{\cdot}53}{2}$ = 56·6825.

Then area of sector A F B C = 279·36
" " of triangle A B C = 56·6825
Gives area of segment A F B = 336·0425. Ans.

3. What is the area of the segment of a circle whose arc is a quadrant, the diameter being 24 perches?

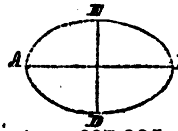
Ans. 41·0976 poles.

Problem 42.

To find the area of an ellipse.

Rule.—Multiply the two axes together, and their product by the decimal ·7854, and the result will be the required area.

1. Required the area of an ellipse whose transure axes A B = 70 feet, and the conjugate axes D E = 50 feet.



$70 \times 50 = 3500 \times {\cdot}7854 = 2748{\cdot}9$ area.

2. What is the area of an ellipse whose axes are 35 and 25? Ans. 687·225.

3. What is the area of an ellipse whose axes are 50 and 45? Ans. 1767·15.

REVIEW.

1. How will you find the area of a square? 2. How will you find the length of the side of a square? 3. When the diagonal is given, how will you find the area of a square? 4. When the area of a square is given, how will you find the diagonal? 5. When the diagonal of a square is given, how will you find the side? 6. How will you cut off a given area from a square parallel to either side? 7. When the length and breadth of a rectangle are given, how will you find the area? 8. When the area and either side of a rectangle are given, how will you find the other side? 9. When the area and the proportion of the two sides of a rectangle are given, how will you find the sides? 10. When the sides of a rectangle are given, how will you cut off a given area parallel to either side? 11. How do you find the area of a rhombus? How will you find the area of a rhomboid? How will you find the area of a parallelogram? 12. When the area of a rhombus or rhomboid, and the length of the side are given, how will you find the perpendicular height? 13. When the base and perpendicular are given, how will you find the area? 14. The three sides of a triangle being given, how will you find the area? 15. When two sides of a right-angled triangle are given, how will you find the third side? Rule second? Rule third? 16. When the sum of the hypotenuse and perpendicular, and the base of a right-angled triangle are given, how will you find the hypotenuse and perpendicular? 17. How will you find the area of an equilateral triangle? 18. When the side of an equilateral triangle is given, how will you find the perpendicular? When the area of an equilateral and the perpendicular are given, how will you find the side? When the area of an equilateral triangle is given, how will you find the side? 19. How will you find the area of an isosceles triangle when the length of the side is given? When the area and base of an isosceles triangle are given, how will you find the length of the equal sides? 20. How will you find the area of a scalene triangle, the base and perpendicular being given? 21. When the area and base of any triangle are given, how will you find the perpendicular height? 22. When the base and perpendicular of any plane triangle are given, how will you find the

side of the inscribed square? 23. How do you find the area of a trapezium? 24. How will you find the area of a trapezoid? 25. How will you find the area of a regular polygon? How will you find the area of a regular polygon when one of its equal sides only is given? How will you find the areas of polygons by the table? 26. When the area of a regular polygon is given, how will you find the side? 27. How will you find the area of an irregular figure bounded on one side by a straight line? 28. How will you find the circumference of a circle when the diameter is given; or the diameter when the circumference is given? 29. How will you find the area of a circle? 30. When the area of a circle is given, how will you find the diameter or circumference? 31. How will you find the area of a circular ring, or the area between two concentric circles? 32. When the diameter or circumference of a circle is given, how will you find the side of an equivalent square? 33. When the diameter or circumference of a circle is given, how will you find the side of an inscribed square? 34. How will you find the diameter of a circle, equal in area to any given superficies? 35. When the diameter of a circle is given, how will you find another containing a proportionate quantity? 36. How will you find the length of a circular arc when the number of degrees and radius are known? 37. How will you find the length of the arc of a circle when the chord and radius are given? 38. When the chord and versed sine are given, how will you find the diameter of a circle? 39. When the versed sine of an arc and the diameter of the circle are given, how will you find the chord? 40. When the chord and versed sine are given, how will you find the area of a sector? 41. How will you find the area of a segment of a circle? 42. How will you find the area of an ellipse?

NOTE.—It is very important that the pupil should pass a strict examination in the *reviews*, by having a certain portion assigned him for a daily lesson, and on no account should this duty be neglected by the instructor.

PART SECOND.

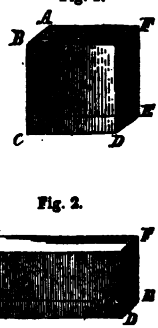
Mensuration of Solids.

DEFINITIONS.

The measure of any solid body is the whole capacity or content of that body, when considered under the triple dimensions of *length*, *breadth*, and *thickness*.

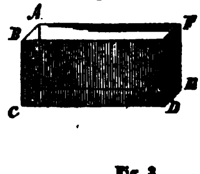
A *cube* whose side is one *inch*, one *foot*, or one *yard*, &c., is called the *measuring unit;* and the content or solidity of any figure is computed by the number of those cubes contained in that figure.

Fig. 1.



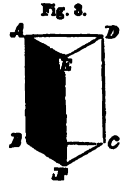
1. A *cube* is a solid contained by six equal square sides; as A B C D E F.

Fig. 2.



2. A *parallelopipedon* is a solid contained by six rectangular plane sides or faces, every opposite two of which are equal and parallel, as A B C D E F.

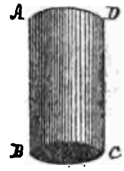
Fig. 3.



3. A *prism* is a solid whose ends are two equal, parallel, and similar plane figures, and its sides parallelograms; as A B C D E F.

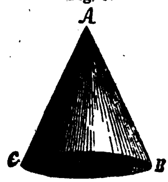
It is called a triangular prism when its ends are triangles; a square prism when its ends are square, &c.

Fig. 4.



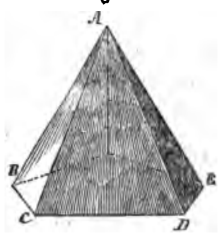
4. A *cylinder* is a solid described by the revolution of a rectangle about one of its sides, as an axis, which remains fixed; as A B C D.

Fig. 5.



5. A *cone* is a solid described by the revolution of a right-angled triangle about one of its legs, which remains fixed; as A B C.

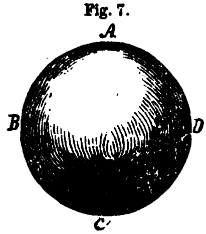
Fig. 6.



6. A *pyramid* is a solid whose sides are all triangles meeting in a point at the vertex, and the base any plane figure; as A B C D E.

When the base is a triangle, it is called a triangular pyramid, &c.

Fig. 7.



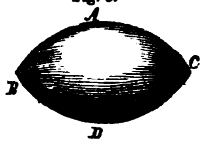
7. A *sphere* is a solid described by the revolution of a semicircle about its diameter, which remains fixed; as A B C D.

The *centre* of a sphere is a point within the figure, equally distant from every part of its convex surface.

A *diameter* of the sphere is a straight line passing through its centre.

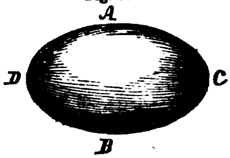
The *axis* of a sphere is any line about which it revolves; and the points at which the axis meets the surface are called the *poles*.

Fig. 8.



8. A *circular spindle* is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed; as A B D C.

Fig. 9.



9. A *speroid* or *ellipsoid* is a solid generated by the revolution of a semi-ellipsis about one of its axes, which remains fixed; as A B D C.

A spheroid is called *prolate* when the revolution is made about the transverse axes, and *oblate* when it is made about the conjugate axes.

10. The *segment* of a pyramid, sphere, or any other solid, is a part cut off from the top of it by a plane parallel to the base of the figure.

11. A *frustum* or *trunk* is a part that remains at the bottom after the segment is cut off.

12. The *zone of a sphere* is that part which is intercepted between the parallel planes; and when those planes are equally distant from the centre, it is called the middle zone of the sphere.

13. The *height* of a solid is a perpendicular drawn from its vertex to the base, or to the plane on which it is supposed to stand.

14. A *wedge* is a solid, having a rectangular base, and two of its opposite sides meeting in an edge.

15. A *prismoid* is a solid, having on its ends two rectangles parallel to each other; and its upright sides are four trapezoids.

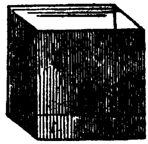
The mensuration of solids is divided into two parts:

1. The mensuration of the surfaces of solids.
2. The mensuration of their solidities.

Problem 1.

To find the area of the surface of a cube.

Rule.—Multiply the square of the length of one side by the number of sides, and the product will be the area of the surface.


Cube.

1. The side of a cube is 18 inches; what is the area of its surface?

Thus, $18 \times 18 = 324 \times 6 = 1944$ in. $\div 144 = 13{\cdot}5$ square feet.

2. The side of a cube is 25 inches; what is the area of the surface? Ans. $26\frac{1}{24}$ feet.

3. The side of a cube is 19 feet; what is the area of its surface? Ans. 2166 feet.

4. The side of a cube is 25·5 feet; what is the area of its surface? Ans. 3901·5 feet.

5. The side of a cube is 36 yards; what is the area of its surface in square feet?

PROBLEM 2.

The area of the surface of a cube being given, to find the length of the side.

RULE.—Divide the area by 6, and extract the square root of the quotient.

1. The area of a cube is 2400 square inches; required the length of the side.

Thus, $2400 \div 6 = \sqrt{400} = 20$ inches. Ans.

2. The area of a cube is 216 square feet; what is the length of the side? Ans. 6 feet.

3. The area of a cube is 5400 square inches; what is the length of the side? Ans. $2\frac{1}{2}$ feet.

4. The area of a cube is 258 square feet; what is the length of the side? Ans. 6·55+.

5. The area of a cube is 1800 square inches; what is the length of the side?

PROBLEM 3.

To find the solidity of a cube, the length of one of the sides being given.

RULE.—Cube the given side.

1. The side of a cube is 25·5 inches; what is the solidity?

Thus, $(25{\cdot}5^3)$ that is, $25{\cdot}5 \times 25{\cdot}5 \times 25{\cdot}5 = 16581{\cdot}375$ cubic inches.

2. The side of a cube is 15 inches; what is the solidity? 1·9531 feet. Ans.

3. The side of a cube is 6 feet; what is the solidity? Ans. 216 feet.

PROBLEM 4.

To find the side of a cube, the solidity being given.

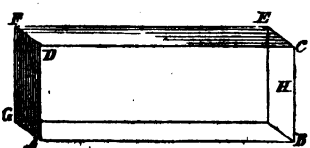
RULE.—Extract the cube root of the solidity.

1. What is the length of the side of a cube containing 36 solid feet? $\sqrt[3]{36} = 3 \cdot 3019$, side required.
2. What is the length of the side of a cube whose solidity is 1800 inches? Ans. 12·1644 inches.
3. What is the length of the side of a cube whose solidity is 789 cubic feet? Ans. 9·2404 feet.
4. What is the length of the side of a cube whose solidity is 2984 yards?

PROBLEM 5.

To find the solidity of a parallelopipedon.

RULE.—Multiply the length by the breadth, and that product again by the depth, or altitude, and it will give the solidity required.



Parallelopipedon.

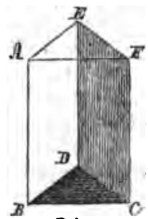
1. Required the solidity of the parallelopipedon A B C D E F G H, whose length A B is 8 feet, its breadth 4·5 feet, and depth or altitude A D, 6·75 feet.
Thus, $AB \times AD \times FD = 8 \times 6 \cdot 75 \times 4 \cdot 5 = 54 \times 4 \cdot 5 = 243$ solid feet. Ans.
2. What is the solidity of a block of marble whose length is 10 feet, breadth 5·75 feet, and depth 3·5 feet?
Ans. 201·25 feet.
3. The length of a parallelopipedon is 36 inches, the width 20 inches, and the depth 18 inches; how many solid feet will it contain? Ans. 7·5 feet.
4. How many bushels are contained in a bin 5·5 feet in length, 4·75 feet in width, and 3·75 in depth?
Ans. 78·724 bushels.
5. What is the solidity of a block of marble whose length is 12 feet, breadth $5\frac{3}{4}$, and depth $2\frac{1}{2}$ feet? Ans. $172\frac{1}{2}$ feet.
6. The length of a parallelopipedon is 15 feet, and each side of its square base 21 inches; what is the solidity?
Ans. 45·9375 feet.

Problem 6.

To find the solidity of a prism.

Rule.—Multiply the area of the base into the perpendicular height of the prism, and the product will be the solidity.

1. What is the solidity of the triangular prism A B C D E F, whose length A B is 20 feet, and either of the equal sides B C, C D, or D B of one of its equilateral ends B C D, 5 feet?



Prism.

See Problem 25, Mensuration of Superficies.

Thus, the area of the base is $5 \times 5 = 25$; and $\cdot433013 \times 25 = 10\cdot825325 \times 20 = 216\cdot5065$ ft. solidity required.

2. What is the solidity of a triangular prism whose length is 18 feet, and one side of the equilateral end 1·5 feet?

Thus, the area of the triangular base is $1\cdot5^2 = 2\cdot25 \times \cdot433013 = \cdot97427925 \times 18 = 17\cdot5370265$. Ans.

3. What is the value of a prism whose height is 32 feet, and each side of the equilateral end 14 inches, at 20 cents per solid foot? Ans. D. 3·772.

4. What is the solidity of a regular pentagonal prism whose altitude is 20 feet, and each side of the base 15 feet?

Thus, $15^2 = 225$; and $1\cdot7204774 \times 225 = 387\cdot107415 =$ the area of the base.

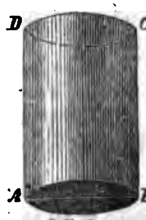
Hence $387\cdot107415 \times 20 = 7742\cdot1483 =$ solidity.

5. What is the number of cubic or solid feet in a regular pentagonal prism of which the altitude is 15 feet, and each side of the base 3·75 feet? Ans. 362·913.

Problem 7.

To find the convex surface of a cylinder.

Rule.—Multiply the circumference or periphery of the base by the height of the cylinder, and the product will be the convex surface required; to which add the area of each, and the sum will be the whole surface of the cylinder.



Cylinder.

1. What is the convex surface of the right cylinder A B C D, whose length B C is 24 feet, and the diameter of its base 16 feet?

Thus, $3\cdot1416 \times 16 = 50\cdot2656$, the circumference of the base.

And $50{\cdot}2656 \times 24 = 1206{\cdot}3744$ square feet, the convex surface required.

2. What is the whole surface of a right cylinder, the diameter of whose base is 2·5 feet, and height 5 feet?

Thus, $3{\cdot}1416 \times 2{\cdot}5 = 7{\cdot}854$, the circumference of the base.

And $7{\cdot}854 \times 5 = 39{\cdot}27$ square feet, the convex surface.

Then to get the whole surface, $2{\cdot}5^2 \times {\cdot}7854 \times 2 = 6{\cdot}25 \times {\cdot}7854 = 4{\cdot}90875 \times 2 = 9{\cdot}8175$ square feet, the area of the ends.

Then $39{\cdot}27 + 9{\cdot}8175 = 49{\cdot}0875$ square feet, the whole surface.

3. What is the convex surface of a cylinder, the diameter of whose base is 20, and altitude 50 feet?

Ans. 3141·6 square feet.

4. What is the convex surface of a cylinder whose base is 30 inches, and altitude 5 feet? Ans. 5654·88 sq. inches.

5. What is the whole surface of a right cylinder, the diameter of whose base is 16 inches, and its length 20 feet?

Ans. 86·5685 feet.

Problem 8.

To find the solidity of a cylinder.

Rule.—Multiply the area of the base by the perpendicular height of the cylinder, and the product will be the solidity.

1. What is the solidity of a cylinder, the diameter of whose base is 40 feet, and altitude 25 feet?

Thus, $40^2 = 1600 \times {\cdot}7854 = 1256{\cdot}64$ = area of the base.

Then $1256{\cdot}64 \times 25 = 31416$ solid feet. Ans.

2. What is the solidity of a cylinder whose height is 5 feet, and the diameter 2 feet? Ans. 15·708 solid feet.

3. What is the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet?

Ans. 2120·58 cubic feet.

4. The length of a cylinder is 30 feet, and the diameter 20 inches; what is the solidity? Ans. 65·45 solid feet.

5. How many solid feet in a round stick of timber 16 feet long, and the diameter at each end 15 inches?

Ans. 19·635 solid feet.

6. Required the solidity of a cylinder, the diameter of whose base is 30 inches, and height 50 inches?

Ans. 20·4531 solid feet.

PROBLEM 9.

To find the whole surface of a right cone.

RULE.—Multiply the circumference of the base by the slant height, or the length of the side of the cone, and half the product will be the area of the convex surface; to which add the area of the base, and the sum will be the whole surface of the cone.

1. What is the convex surface of the cone whose vertex is C, the diameter A D of its base being 8·5 feet, and the side C A, 50 feet?

Cone.

Thus, first $3{\cdot}1416 \times 8{\cdot}5 = 26{\cdot}7036$ = circumference of base.

Then $26{\cdot}7036 \times 50 \div 2 = 667{\cdot}59$, convex surface.

2. The diameter of a cone is 4·5 feet, and the slant height 20 feet; required the convex surface. Ans. 141·372.

3. The diameter of the base of a cone is 3 feet, and the slant height 15 feet; what is the convex surface? Ans. 70·686 sq. ft.

4. The slant height of a cone is 20 feet, and diameter 3 feet; required the surface of the cone.

Ans. 101·3166 square feet.

5. The circumference of the base of a cone is 10·75, and the slant height is 18·25; what is the entire surface?

Ans. 107·29021 square feet.

PROBLEM 10.

To find the solidity of a cone.

RULE.—Multiply the square of the diameter of the base by ·7854, and that product by *one-third* of the perpendicular altitude; the product will be the solidity.

1. Required the solidity of a cone, the diameter of whose base is 18 inches, and its altitude 15 feet.

Thus, 18 in. $= 1{\cdot}5^2 \times {\cdot}7854 = 1{\cdot}26715$ = area of base.

And $176715 \times \frac{15}{3}\ (5) = 8{\cdot}8357$ feet, solidity required.

2. What is the solidity of a cone, the *area* of whose base is 380 square feet, and altitude 48 feet?

Thus, $380 \times 48 = 18240 \div 3 = 6080$ feet. Ans.

3. Required the solidity of a cone whose altitude is 10½ feet, and the circumference of its base 9 feet.

Ans. 22·5609 cubic feet.

4. The circumference of the base of a cone is 40 feet, and the height 50 feet; required the solidity.

Thus, $40^2 \times \cdot07958 = 1600 \times \cdot07958 = 127\cdot328 =$ area of base.

Then $127\cdot328 \times \frac{50}{3} = \frac{6366\cdot4}{4} = 2122\cdot1333$ feet. Ans.

5. The circumference of the base of a cone is 10 feet, and the perpendicular altitude 12 feet; what is the solidity?

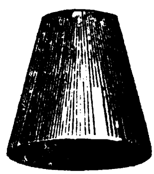
Ans. 31·829 cubic feet.

Problem 11.

To find the surface of the frustum of a right cone.

Rule.—Add together the circumferences of the two bases, and multiply the sum by half the slant height of the frustum, and the product will be the convex surface, to which add the areas of the bases, and the entire surface is acquired.

1. What is the convex surface of the frustum of a cone, of which the slant height is $12\frac{1}{2}$ feet, and the circumferences of the bases 8·4 and 6 feet?



Cone.

Thus, $8\cdot4 + 6 = 14\cdot4$; half side, $6\cdot20 \times 14\cdot4 = 90$ square feet. Ans.

2. What is the convex surface of the frustum of a cone, the circumference of the greater base being 30 feet, and of the less 10 feet; the slant height being 20 feet? Ans. 400 square feet.

3. Required the entire surface of the frustum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet? Ans. 439·824 square feet.

Problem 12.

To find the solidity of the frustum of a cone.

Rule.—Add together the areas of the two ends and geometrical mean between them.

Multiply this sum by *one-third* of the altitude, and the product will be the solidity.

1. How many cubic feet in the frustum of a cone whose altitude is 26 feet, and the diameters of the bases 22 and 18 feet?

Thus, $22^2 \times \cdot7854 = 380\cdot134$, area of lower base.

$18^2 \times \cdot7854 = 259\cdot47 =$ area of upper base.

Then $\sqrt[2]{380{\cdot}134 \times 254{\cdot}47} = 311{\cdot}018$ = mean.

And $(380{\cdot}134 + 254{\cdot}47 + 311{\cdot}018) \times \frac{26}{3} = 8195{\cdot}39$. Ans.

2. What is the solidity of the frustum of a cone, the altitude being 18, the diameter of the lower base 8, and the upper base 4? Ans. 527·7888.

3. How many cubic feet in a piece of round timber, the diameter of the greater end being 18 inches, and that of the less 9 inches, and the length 14·25 feet? Ans. 14·68943 feet.

4. What is the solidity of the frustum of a cone, the diameter of the greater end being 4 feet, that of the less end 2, and the altitude 9 feet? Ans. 65·9736 feet.

5. What is the solidity of the frustum of a cone, the circumference of the greater end being 20 feet, and that of the less end 10 feet, and the height 21 feet? Ans. 389·942 feet.

Problem 13.

The solidity and altitude of a cone being given, to find the diameter.

Rule.—Divide the solidity by the product of ·7854 and one-third of the altitude, and the square root of the quotient will be the diameter.

1. The solidity of a cone is 16 feet, and the altitude 9 feet; what is the diameter?

Thus, $\cdot 7854 \times 3 = 2{\cdot}3562$; $16 \div 2{\cdot}3562 = \sqrt[2]{6{\cdot}7906} = 2{\cdot}6057$ feet. Ans.

2. The solidity of a cone is 18 feet, and the altitude 8 feet; required the diameter. Ans. 2·9315 feet.

Problem 14.

The solidity and diameter of a cone being given, to find the altitude.

Rule.—Divide the solidity by the product of ·7858, and the square of the diameter, and the quotient, being multiplied by 3, will give the altitude.

1. The solidity of a cone is 30 feet, and the diameter 2 feet; what is the altitude?

Thus, $2 \times 2 = 4$; $\cdot 7854 \times 4 = 3{\cdot}1416$; $30 \div 3{\cdot}1416 = 9{\cdot}5492 \times 3 = 28{\cdot}6476$ feet. Ans.

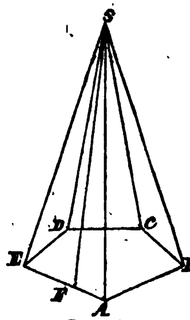
2. The solidity of a cone is 2513·28 feet, and the diameter 20 feet; what is the altitude? Ans. 24 feet.

Problem 15.

To find the surface of a regular pyramid.

Rule.—Multiply the perimeter of the base by half the slant height, and the product will be the convex surface; to this add the area of the base, if the entire surface is required.

1. In the regular pentagonal pyramid S—A B C D E, the slant height S F is equal to 45, and each side of the base is 15 feet; required the convex surface, and also the entire surface.



Pyramid.

Thus, $15 \times 5 = 75 =$ perimeter of the base; $75 \times 22\frac{1}{2}$ ($\frac{1}{2}$ 45) $= 1687·5$ square ft. = area of convex surface.

And $15^2 = 225$; then $225 \times 1·7204774 = 387·107415 =$ the area of the base.

See Table, Prob. 25, Men. of Super.

Hence convex surface,	1687·5
Area of the base,	387·107415
Entire surface, Ans.	2074·607415 square feet.

2. What is the entire surface of a regular pyramid whose slant height is 15 feet, and the base a regular pentagon, of which each side is 25 feet? Ans. 2012·798 square feet.

3. What is the entire surface of a regular octagonal pyramid, of which each side of the base is 9·941 yards, and the slant height 15? Ans. 1078·628 square yards.

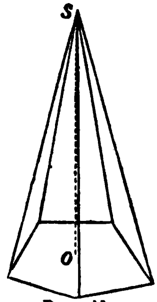
4. Required the whole surface of a triangular pyramid, each side of its base being $5\frac{1}{2}$ feet, and its slant height $17\frac{1}{2}$ feet.
Ans. 157·4736 square feet, the whole surface.

5. Required the outward surface of a triangular pyramid, each side of its base being $3\frac{1}{2}$ feet, and its slant height 14 feet.
Ans. 73·5 feet.

Problem 16.

To find the solidity of a pyramid.

Rule.—Multiply the area of the base by the altitude, and divide the product by 3; the quotient will be the solidity.



Pyramid.

1. What is the solidity of a pyramid, the area of whose base is 215 square feet, and the altitude S O = 45 feet?

Thus, $215 \times 45 = 9675 \div 3 = 3225$ solid feet. Ans.

2. How many solid yards are there in a triangular pyramid whose altitude is 90 feet, and each side of its base 3 yards?

Ans. 38·97114 yards.

3. What is the solidity of a regular pyramid, its altitude being 12 feet, and each side of its base 2 feet? Ans. 27·5276 solid feet.

4. Required the solidity of a triangular pyramid whose height is 30 feet, and each side of the base 3 feet.

Ans. 38·97117 feet.

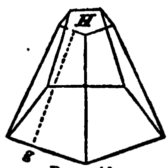
5. Required the solidity of a square pyramid, each side of whose base is 30, and perpendicular height 20.

Ans. 6000

Problem 17.

To find the convex surface of the frustum of a pyramid.

Rule.—Multiply the sum of the perimeters of the two bases by the slant height of the frustum, and the product will be the convex surface.



Pyramid.

1. In the frustum of the regular pentagonal pyramid, each side of the lower base is 30, and each side of the upper base is 20 feet, and the slant height g H is equal to 15 feet; what is the convex surface of the frustum?

Thus, $30 \times 5 = 150$; $20 \times 5 = 100 + 150 = 250 \div 2 = 125 \times 15 = 1875$ square feet. Ans.

2. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 55 feet, each side of the lower base 8 feet, and each side of the upper base 4 feet?

Ans. 2310 square feet.

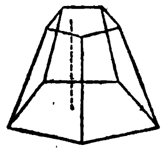
Problem 18.

To find the solidity of the frustum of a pyramid.

Rule.—Add together the areas of the two bases of the frustum of a geometrical mean proportional between them; and then

multiply the sum by the altitude, and take one-third of the product for the solidity.

1. What is the solidity of the frustum of a pentagonal pyramid, the area of the lower base being 16 feet, and of the upper base 9 square feet, and altitude 7 feet?



Pyramid.

Thus, $16 \times 9 = 144$; $\sqrt[2]{144} = 12$ the mean, area of the lower base $= 16$

9 upper base.
12 mean of bases.
37 sum.

$37 \times 7 = 259 \div 3 = 86\frac{1}{3}$ solid feet. Ans.

2. What is the content of a regular hexagonal frustum whose height is 6 feet, the side of the greater end 18 inches, and of the less end 12 inches? Ans. 24·681724 cubic feet.

3. How many cubic feet in a square piece of timber, the areas of the two ends being 504 and 372 inches, and the length $31\frac{1}{2}$ feet? Ans. 95·447.

4. What is the solidity of the frustum of a square pyramid, one side of the greater end being 18 inches, that of the less end 15 inches, and the height 60 inches?

Thus, $\frac{(18^2 + 15^2 + 18 \times 15)}{3} \times 60 = 819 \times 60 \div 3 = 16380$ inches, solidity. Ans.

5. The height of the frustum of a square pyramid is 8 feet each side, the base 16 inches, and the top 10 inches; required the solidity of the frustum. Ans. 9·55 cubic feet.

Problem 19.

To find the solidity of a wedge.

Rule.—Add the length of the edge to twice the length of the base, and multiply the sum by the height of the wedge, and that product by the breadth of the base, and $\frac{1}{6}$ of the last product will be the solidity.

Wedge.

1. The length and breadth of the base of a wedge are 35 and 15 inches, and the length of the edge is 55 inches; what is the solidity, supposing the height to be 17·14508 inches?

Thus, $35 \times 2 + 55 = 125$; then $\frac{17{\cdot}14508 \times 15 \times 125}{6} =$

$\frac{32147 \cdot 025}{6} = 5357 \cdot 8375$ cubic inches $\div 1728 = 3 \cdot 1006$ feet. Answer.

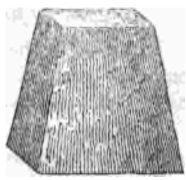
2. If the base of a wedge be 27 inches and 8 inches, the edge 36 inches, and the height 3·5 feet, what is the solidity?

Ans. 2·9166 cubic feet.

Problem 20.

To find the solidity of a prismoid.

Rule.—To the sum of the areas of the two ends add four times the area of the section parallel to, and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the height will give the solidity.



Prismoid.

1. What is the solidity of a rectangular prismoid, the length and breadth of one end being 12 and 8 inches, and the corresponding sides of the other 8 and 6 inches, and the perpendicular height 60 inches?

Thus, $12 \times 8 = 96$; $8 \times 6 = 48 + 96 = 144$, sum of the areas of the two ends.

Then $12 + 8 = 20 \div 2 = 10$, length of the middle rectangle.

$8 + 6 = 14 \div 2 = 7$, breadth of the middle rectangle.

Hence $4 \times 10 \times 7 = 280$; $4 \times 70 = 280 = 4$ times the area of the middle rectangle.

$144 + 280 \times \frac{60}{6} = 424 \times 10 = 4240$ cubic inches.

And $4240 \div 1728 = 2 \cdot 4537$ feet. Ans.

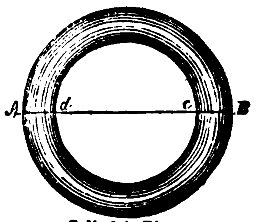
2. What is the solidity of a stick of hewn timber whose ends are respectively 30 by 27 inches, and 24 by 18 inches, and whose length is 48 feet?

Ans. 204 feet.

Problem 21.

To find the convex superficies of a cylindric ring.

Rule.—To the thickness of the ring add the inner diameter, and this sum being multiplied by the thickness, and the product again by 9·8696, (or the square of 3·1416,) will give the superficies required.



Cylindric Ring.

1. The thickness A d of the cylindric ring, is 3 inches, and the inner diameter c d, 12 inches; what is the convex surface?

Thus, $12 + 3 = 15 \times 3 = 45 \times 9{\cdot}8696 = 444{\cdot}132$ square inches. Ans.

2. The thickness of a cylindric ring is 2 inches, and the inner diameter 18 inches; required the convex superficies. Ans. 394·784 inches.

3. The thickness of a cylindric ring is 2 inches, and the inner diameter 12 inches; what is the convex superficies?

Ans. 276·3488 inches.

Problem 22.

To find the solidity of a cylindric ring.

Rule.—To the thickness of the ring add the inner diameter, and this sum being multiplied by the square of half the thickness, and the product again by 9·8696, will give the solidity.

1. What is the solidity of an anchor ring whose inner diameter is 8 inches, and thickness of metal 3 inches?

Thus, $8 + 3 = 11$; $1{\cdot}5^2 = 2{\cdot}25 \times 11 = 24{\cdot}75 \times 9{\cdot}8696 = 244{\cdot}2726$ cubic inches. Ans.

2. The inner diameter of a cylindric ring is 12 inches, and its thickness 4 inches; what is its solidity?

Ans. 631·6544 inches.

3. Required the solidity of a cylindric ring whose inner diameter is 12 inches, and thickness 5 inches.

Ans. 1048·645 inches.

4. What is the solidity of an anchor ring whose inner diameter is 9 inches, and the thickness of metal 3 inches?

Ans. 266·4792 inches.

Problem 23.

The solidity and thickness of a cylindric ring being given, to find the inner diameter.

Rule.—Divide the solidity of 9·8696, and that quotient by the square of half the thickness; from which subtract the thickness, and the remainder will be the inner diameter of the ring.

1. The thickness of a cylindric ring is 4 inches, and its solidity 789·568 solid inches; what is its inner diameter?

Thus, $789{\cdot}568 \div 9{\cdot}8696 = 80 \div 2^2 = (4) = 20 - 4 = 16$ inches, diameter.

2. What is the inner diameter of a cylindric ring whose solidity is 1 solid foot, and thickness 4 inches? Ans. 30·77 in.

3. What is the inner diameter of a cylindric ring whose solidity is 244·2726 inches, and thickness 3 inches. Ans. 8 in.

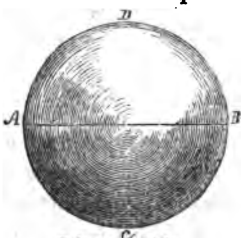
Problem 24.

To find the convex surface of a sphere.

Rule.—Multiply the diameter of the sphere by its circumference, and the product will be the convex superficies required. The curve surface of any zone or segment will also be found by multiplying its height by the whole circumference of the sphere.

1. What is the convex surface of a sphere or globe A D B C, whose diameter A B is 16 inches?

Thus, $3{\cdot}1416 \times 16 = 50{\cdot}2656 \times 16 = 804{\cdot}2496$ square inches, the surface required.



Sphere, or Globe.

2. What is the convex superficies of a sphere whose diameter is $1\frac{1}{3}$ feet, and the circumference 4·1888 feet?

Thus, $1\frac{1}{3} = \frac{4}{3}$, and $\frac{4}{3} \times 4{\cdot}1888 = 16{\cdot}7552 \div 3 = 5{\cdot}58506$ feet. Ans.

3. If the diameter or axis of the earth be $7957\frac{3}{4}$ miles, what is the whole surface, supposing it to be a perfect sphere?

Thus, $7957{\cdot}75 \times 3{\cdot}1416 = 25000{\cdot}0674$, the circumference; $7957{\cdot}75 \times 25000{\cdot}0674 = 198944286{\cdot}35235$ sq. miles. Ans.

4. What is the area of the convex surface of a globe whose diameter is 4 feet? Ans. 50·2656 square feet.

5. What is the convex surface of a sphere whose diameter is 6 feet? Ans. 113·0976 feet.

Problem 25.

To find the solidity of a sphere or globe.

Rule.—Multiply the cube of the diameter by ·5236, and the product will be the solidity.

1. What is the solidity of a sphere whose diameter is $1\frac{1}{3}$ feet?

Thus, $1\frac{1}{3} = \frac{4^3}{3} \times \cdot5236 = \frac{64}{27} \times \cdot5236 = 33\cdot5104 \div 27 =$ $1\cdot2411$ feet. Ans.

2. What is the solidity of the earth, supposing it to be perfectly spherical, and the diameter 7957·75 miles?

Thus, $7957\cdot75^3 \times \cdot5236 = 63325785\cdot0625 \times 7957\cdot75 \times \cdot5236$ $= 503930766081\cdot109375 \times \cdot5236 = 263858149120\cdot06886875$. Answer.

3. What is the solidity of a globe whose diameter is 3 feet 4 inches? Ans. 19·3926 cubic feet.

4. What is the solidity of a globe whose diameter is 17 inches? Ans. 1·4493 feet.

5. What is the solidity of a sphere whose diameter is 6? Ans. 113·0976.

6. What is the solidity of a sphere whose diameter is 10? Ans. 4188·8.

Problem 26.

The convex surface of a globe being given, to find its diameter.

Rule.—Multiply the given area by 31831, and the square root of the product will be the diameter.

1. What is the diameter of that globe, the area of whose convex surface is 14 square feet?

Thus, $14 \times \cdot31831 = \sqrt{4\cdot45634} = 2\cdot1110$ feet, the diameter required.

2. The convex surface of a sphere is 1 square rood; required the diameter. Ans. 3·5682 rods.

3. The expense of gilding a ball, at D. 1·80 per square foot, is 34 dollars; required the diameter. Ans. 2·452 feet.

Problem 27.

The solidity of a globe being given, to find the diameter.

Rule.—Divide the solidity by ·5236, and extract the cube root of the quotient.

1. The solidity of a globe is 2000 solid inches; what is the diameter?

Thus, $2000 \div \cdot5236 = \sqrt{3819\cdot7097} = 15\cdot631$ inches. Ans.

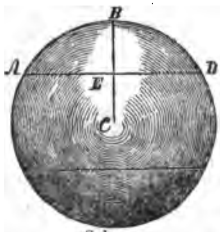
2. The solidity of a globe is 10 solid feet; required the diameter. Ans. 2·67 feet.

PROBLEM 28.

To find the convex surface of a spherical zone.

RULE.—Multiply the height of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface.

1. What is the convex surface of the zone A B D, the height B E being 9 inches, and the diameter of the sphere 42 inches?



Sphere.

Thus, 42 × 3·1416 = 131·9472 = circumference × 9 = 1187·5248 square inches, surface. Ans.

2. The diameter of a sphere is 21 inches; what is the surface of a zone whose height is 4½ inches?

Ans. 296·8812 sq. inches.

3. The diameter of a sphere is 25 feet, and height of the zone 4 feet; what is the surface of the zone?

Ans. 314·16 square feet.

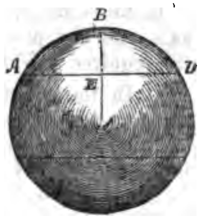
PROBLEM 29.

To find the solidity of a spherical segment with one base.

RULE.—To 3 times the square of the radius of the base add the square of the height.

Multiply this sum by the height, and the product by ·5236; the result will be the solidity of the segment.

1. What is the solidity of the segment A B D, the height B E being 4 feet, and the diameter A D of the base being 14 feet?



Sphere.

Thus, 7 × 7 = 49 × 3 = 147 + (4^2 = 16) = 163 × 4 × ·5236 = 341·3872 solid feet, which is the solidity of the segment A B D.

2. What is the solidity of a spherical segment, the diameter of its base being 17·23368, and height 4·5? Ans. 572·5566.

3. What is the solidity of a segment, when the diameter of the sphere is 20, and the altitude of the segment 9 feet?

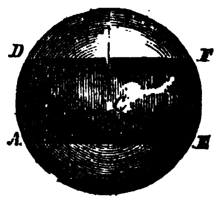
Ans. 1781·2872 cubic feet.

PROBLEM 30.

To find the solidity of a spherical segment having two bases.

RULE.—To the sum of the squares of the radii of the two bases add one-third of the square of the distance between them; then multiply this sum by the breadth, and the product by 1·5708, and the result will be the solidity.

1. Required the solid content of the zone A D F E, the diameter of whose greater base A E is equal to 20 inches, and the less diameter D F 15 inches, and the distance between the two bases 10 inches.



Sphere.

Thus, $10^2 = 100$; $75^2 = 56{\cdot}25$; $100 \div 3 = 33{\cdot}33$; $100 + 56{\cdot}25 + 33{\cdot}33 = 189{\cdot}58 \times 10 \times 1{\cdot}5708 = 2977{\cdot}92264$ solid inches. Ans.

2. What is the solidity of the middle zone of a sphere, the diameter of whose greater base is 24 inches, the less diameter 20 inches, and the distance between the bases 4 inches?

Ans. 1566·6112 solid inches.

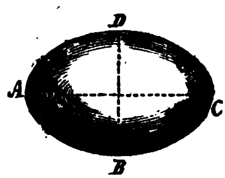
PROBLEM 31.—THE SPHEROID.

The form of the earth is an oblate spheroid, the axis about which it revolves being about 34 miles shorter than the diameter perpendicular to it.

To find the soliditg of an ellipsoid.

RULE.—Multiply the fixed axis by the square of the revolving axis, and the product by the decimal ·5236; the result will be the required solidity.

1. In the prolate spheroid A B C D, the transure or fixed axis A C, is 3 feet, and the conjugate or revolving axis D B, is 2 feet; what is the solidity?



Prolate Spheroid.

Thus, $2^2 = 4 \times 3 = 12 \times {\cdot}5236 = 6{\cdot}2832$ feet, the solidity. Ans.

2. What is the solidity of a prolate spheroid whose fixed axis is 100, and revolving axis 6 feet? Ans. 1884·96.

3. What is the solidity of an oblate spheroid whose axes are 20 and 10, ($20^2 \times 10 \times {\cdot}5236$.) Ans. 2094·4.

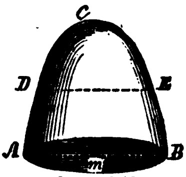
PROBLEM 32.

To find the solidity of a paraboloid.

RULE.—Multiply the area of the base by half the altitude, and the product will be the solidity.

1. What is the solidity of a paraboloid, A C B A, whose height C m is 7 feet, and the diameter A B, its circular base, 4 feet?

Thus, $4^2 \times \cdot 7854 \times (7 \div 2) = 16 \times \cdot 7854 \times 2\cdot 5 = 12\cdot 5664 \times 3\cdot 5 = 43\cdot 9824$ cubic feet. Ans.



Paraboloid.

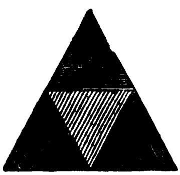
2. Required the solidity of a paraboloid whose height is 50 inches, and the diameter of its base 40 inches. Ans. $10\frac{9}{12}$ ft.

3. Required the solidity of a paraboloid whose height is 50 inches, and the diameter at its base, 100 inches.

Ans. 113·6284 feet.

THE FIVE REGULAR BODIES.

Fig. 1.


Pyramid unfolded.

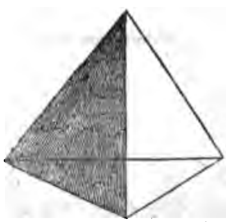
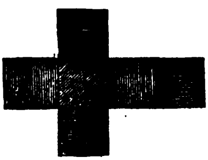

Pyramid.

Fig. 2.


Cube unfolded.

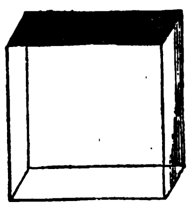
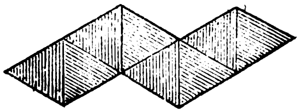
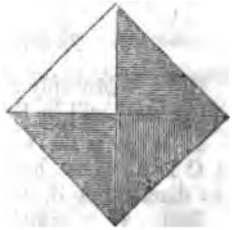

Cube.

Fig. 3.

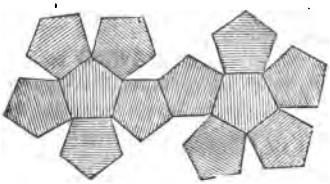


Octaedron unfolded.

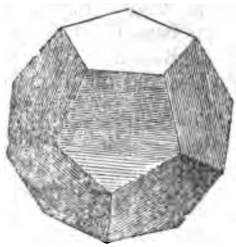


Octaedron.

Fig. 4.

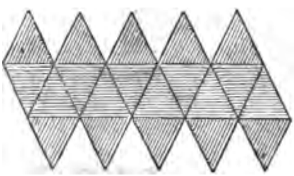


Dodecaedron unfolded.

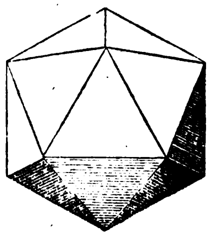


Dodecaedron.

Fig. 5.



Icosaedron unfolded.



Icosaedron.

1. The *Tetraedron*, or equilateral pyramid, is a solid bounded by four equal triangles.

2. The *Hexaedron*, or cube, is a solid bounded by six equal squares.

3. The *Octaedron* is a solid bounded by eight equal triangles.

4. The *Dodecaedron* is a solid bounded by twelve equal pentagons.

5. The *Icosaedron* is a solid bounded by twenty equal triangles.

The above five regular bodies may be represented by making the figures of pasteboard, and cutting the lines half through, so that the parts may be folded and sewed together.

The following table shows the surface and solidity of each of the regular solids, when the linear edge is unity.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1·73205	0·11785
6	Hexaedron	6·00000	1·00000
8	Octaedron	3·46410	0·47140
12	Dodecaedron	20·64578	7·66312
20	Icosaedron	8·66025	2·18169

PROBLEM 33.

To find the surface of a regular solid when the length of the linear edge is given.

RULE.—Multiply the square of the linear edge by the tabular number in the column of surfaces, and the product will be the surface required.

1. The linear edge of a tetraedron is 3; what is its surface?

The tabular area is 1·73205; then $3^2 = 9$, and $1·73205 \times 9 = 15·58845$. Ans.

2. The linear edge of an octaedron is 5; what is its surface?

The tabular area is $3·46410 (\times 5^2 = 25) = 86·6025$ = surface.

3. The linear edge of an icosaedron is 6; what is its surface?

The tabular area is $8·66025 \times (6^2 = 36) = 311·769$ = surface.

PROBLEM 34.

To find the solidity of a regular solid when the length of the linear edge is known.

RULE.—Multiply the cube of the linear edge by the tabular number in the column of solidities, and the product will be the solidity required.

1. What is the solidity of a regular tetraedron whose side is 6? The tabular number is 0·11785 × (6^3 = 216) = 25·4556. Answer.

2. What is the solidity of a regular octaedron whose linear edge is 8?
The tabular number is 0·47140 × (8^2 = 512) = 241·3568. Answer.

3. What is the solidity of a regular dodecaedron whose linear edge is 3?
The tabular number is 7·66312 × (3^3 = 27) = 206·90424 = solidity.

4. Required the solidity of an octaedron whose side is 10 feet.
Ans. 471·4 feet.

5. Required the solidity of a dodecaedron whose side is 4 feet.
Ans. 490·4396 feet.

6. Required the solidity of an icosaedron whose side is 3 feet.
Ans. 58·9056 feet.

7. What is the solidity of a dodecaedron whose side is 9 feet?
Ans. 5586·4144 feet.

Method of finding the decimal *numbers used in this volume.*

When the diameter of a circle is 1, the circumference is 3·1416÷ or 3·141592.	12	·2618 = $\frac{1}{12}$	·7854		
	8	·3927 = $\frac{1}{8}$		8	0·98175 $\frac{1}{8}$
	7	·4488 = $\frac{1}{7}$		7	0·1122 $\frac{1}{7}$
	6	·5236 = $\frac{1}{6}$		6	0·13423 $\frac{1}{6}$
	5	·6283 = $\frac{1}{5}$		5	0·15708 $\frac{1}{5}$
	4	·7854 = $\frac{1}{4}$		4	0·19655 $\frac{1}{4}$
	3	1·0472 = $\frac{1}{3}$		3	0·2618 $\frac{1}{3}$
	2	1·5708 = $\frac{1}{2}$		2	0·3927 $\frac{1}{2}$
	$\frac{3}{4}$	2·2762 = $\frac{3}{4}$		$\frac{1}{4}$	0·1965 $\frac{1}{4}$

$3{\cdot}1416^2 = 9{\cdot}8696$; $\cdot 031832 \div 4 = \cdot 07958$; $\cdot 7854 \div \frac{8}{15} = \cdot 418879$; $\cdot 07958 \div 2 = \cdot 2821$; $1. \div 3{\cdot}1416 = \cdot 031832$; $\sqrt[2]{\cdot 7854} = \cdot 8862$; $\sqrt[2]{\cdot 05} = \cdot 7071$; ·433013 = area of an equilateral triangle.

NOTE.—In the above table, the *exact* products or remainders cannot always be obtained, but by *approximation* the result will be sufficiently correct for all purposes of practical utility. In some cases the circumference is given, 3·141592: this will make

a small difference in the quotient; but the *given* decimal in the *rule* should be used in the solution of questions under *that* rule.

REVIEW.

DEFINITIONS.—1. What is a cube? 2. What is a parallelopipedon? 3. What is a prism? 4. What is a cylinder? 5. What is a cone? 6. What is a pyramid? 7. What is a sphere? What is the centre of a sphere? What is the diameter of a sphere? The axis of a sphere? The poles of a sphere? 8. What is a circular spindle? 9. What is a spheroid or ellipsoid? When is it called *prolate;* when *oblate?* 10. What is the segment of a sphere? 11. What is the frustum of a sphere? 12. What is the zone of a sphere? 13. What is the height of a solid? 14. What is a wedge? 15. What is a prismoid? Into how many parts is the mensuration of solids divided? Name them? (1.) (2.)

PROBLEMS.—1. How will you find the area of the surface of a cube? 2. How will you find the length of the side of a cube? 3. How will you find the solidity of a cube? 4. When the solidity is given, how will you find the side of a cube? 5. How will you find the solidity of a parallelopipedon? 6. How will you find the solidity of a prism? 7. How will you find the convex surface of a cylinder? 8. How will you find the solidity of a cylinder? 9. How will you find the surface of a right cone? 10. How will you find the solidity of a cone? 11. How will you find the frustum of a right cone? 12. How will you find the solidity of the frustum of a cone? 13. How will you find the diameter of a cone, the solidity and altitude being given? 14. How will you find the altitude of a cone, the solidity and diameter being given? 15. How will you find the surface of a regular pyramid? 16. How will you find the solidity of a pyramid? 17. How will you find the convex surface of the frustum of a pyramid? 18. How will you find the solidity of the frustum of a pyramid? 19. How will you find the solidity of a wedge? 20. How will you find the solidity of a prismoid? 21. How will you find the convex surfaces of cylindric rings? 22. How will you find the solidity of a cylindric ring? 23. How will you find the diameter of a cylindric ring? 24. How will you find the convex surface of a sphere? 25. How will you find the solidity of a sphere or globe? 26. How will you find the diameter of a sphere when the convex surface is given? 27. How will you find the diameter of a globe, the solidity being given? 28. How will you find the convex surface of a spherical zone? 29. How will you find the solidity of a spherical segment of one base? 30. How will you find the solidity of a spherical segment having two bases? 31. How will you find the solidity of an ellipsoid? 32. How will you find the solidity of a paraboloid? Name the five regular bodies. 33. How will you find the surface of regular solids? 34. How will you find the solidity of regular solids? How can you find the *decimal* numbers used in mensuration?

Appendix.

Artificers' Work.

Artificers estimate or compute the value of their work by different measures, tables of which have been previously given; but the best method of taking the dimensions of all sorts of artificers' work is by feet, tenths, and hundredths—in other words, by *decimals;* glazing, and masons' flat work, &c., by the foot; painting, plastering, paving, &c., by the yard; flooring, partitioning, roofing, tiling, &c., by the square of 100 feet; brick work, &c., by the rod or perch of 16½ feet, whose square is 272·25 feet. In calculating the square feet, it is usual to omit the ·25, but the more correct way is to use the perfect number (272·25). The practice has formerly been to estimate the various kinds of mechanical work according to the rules and regulations of the several countries of Europe, particularly *England and Germany;* but the method has in some respects been departed from, which has rendered the calculations different in several parts of the United States. The general system of computation will be given, from which any others may easily be derived.

The Carpenter's Rule.

The carpenter's or sliding rule is used in the measurement of timber, boards, and artificers' work; it consists of two equal pieces of box-wood, each one foot long, and connected together by a folding joint; the division of one face of the rule is in inches, half inches, quarter inches, eighths of inches, and sixteenths of inches, and, when the rule is open, is 24 inches or 2 feet in length. The edge of the rule is divided *decimally*, that is, each foot is divided into ten equal parts, and each of those into ten equal parts; consequently the divisions are hundredths of a foot, numbering from the right to the left. By the decimal division of the rule, it is convenient in bringing the dimensions

into integers and decimals, and then, if multiplication is required, it is more easily performed than when expressed in feet and inches. A correct knowledge of the *use* of the rule can only be acquired by *practice*. There are several kinds of rules, or scales for measuring, such as a scale or table of board measure, one of timber measure, another for showing what length for any breadth will make a square foot, &c.

TIMBER MEASURE.

To find the area of a board or plank.

RULE.—Multiply the length by the breadth, and the product will be the content required. Or, multiply the length by the breadth in *inches*, and divide the product by 12.

When the board is broader at one end than at the other, add both ends together, and take half the sum for a mean breadth, and multiply as above directed.

1. Required the content of a board measuring 12 feet 6 inches in length, and 1 foot 3 inches in breadth.

Thus, $12{\cdot}5 \times 1{\cdot}25 = 15{\cdot}625$ feet. Ans.

Or, $12{\cdot}5 \times 15 = 187{\cdot}5 \div 12 = 15{\cdot}625$. Ans.

2. What is the area of a board whose length is 8 feet 6 inches, and breadth 1 foot 3 inches?

By Duodecimals.	*By Decimals.*
Ft. 8 6 in.	Ft. 8 6 in. $= 8{\cdot}5$ feet.
1 3	1 3 $= 1{\cdot}25 \times$
8 6′	$10{\cdot}625$ square feet.
2 1 6″	
10 7 6″ Ans.	

3. How many square feet in a board whose breadth at one end is 15 inches, at the other 17 inches, the length being 6 feet?
Ans. 8 feet.

4. What is the value of a plank whose breadth at one end is 2 feet, and at the other 4 feet, the length being 12 feet, at 10 cents per square foot? Ans. D. 3·60.

5. Required the content of a board whose length is 18 feet 9 inches, and breadth 16 inches. Ans. 25 feet.

6. What is the value of a board whose length is 24 feet, and mean breadth 3 feet 3 inches, at $3\frac{1}{4}$ cents per square foot?
Ans. D. 2·535.

7. Required the content of a board whose length is 20·25 feet, and breadth 1·75 feet.

To find the content of a board or piece of timber whose thickness is more than one inch.

RULE.—Multiply the area of the end in inches by the length, and divide the product by 12; or multiply the area of the end in inches by the length in inches, and divide the product by 144.

1. What is the content of a piece of scantling measuring $18\frac{1}{2}$ feet in length, 6 inches in breadth, and 4 inches in thickness?

Thus, $6 \times 4 = 24 \times 18{\cdot}5 = 444 \div 12 = 37$ feet. Ans.

Or, $18\frac{1}{2} = 222$ in. $\times 24 = 5328 \div 144 = 37$ feet.

2. What number of feet, board measure, is contained in a beam whose length is 42 feet 4 inches, breadth 14 inches, and thickness 12 inches? Ans. $592\frac{2}{3}$ feet.

3. What is the value of a plank whose length is 16·5 feet, breadth 2·25 feet, and thickness 3 inches, at 5 cents per square foot?

Having one dimension of a plank or board given, to find the other dimension, so that the plank shall contain a given area.

RULE.—Divide the given area by the given dimension, and the quotient will be the other dimension.

1. The length of a board is 16 feet; what must be its width that it may contain 12 square feet?

Thus, $16 \times 12 = 192$ inches; 12 square feet $= 144 \times 12 = 1728$ square inches; then $1728 \div 192 = 9$ inches, the width of the board.

2. If a board is 8 inches wide, what length of it will make 4 square feet? Ans. 6 feet.

3. What is the content of a board whose length is 5 feet 7 inches, and breadth 1 foot 10 inches?

Ans. 10 ft. 2′ 10″.

To find the solid content of squared or four-sided timber.

When the ends are equal squares.

RULE.—Multiply the length by the breadth, and that product by the thickness, and the result will be the solid content. To change solid feet into superficial feet, multiply by 12.

1. A square piece of timber is 15 inches broad, 15 inches deep, and 18 feet long; how many solid feet does it contain?

Thus, $15 \times 15 \times 18 = 28{\cdot}125$. Ans.

2. A squared piece of timber is 16 inches broad, 16 inches

deep, and 18 feet long; how many solid and superficial feet does it contain? Ans. 32 solid feet, 384 superficial feet.

3. The length of a piece of timber is 24·5 feet; its ends are equal squares, whose sides are each 1·04 feet; what is the solidity? Ans. 26·4992 solid feet.

4. What is the content of a piece of square timber 40 feet in length, and 18 inches in width and depth?

When the ends are unequal squares.

RULE.—Add together the areas of the two ends, and the product of the dimensions of the ends. Multiply this sum by the length, and one-third of the product will be the solidity.

1. In a piece of timber whose ends are unequal squares, the side of the greater is 18 inches, and that of the less 15 inches, and the length 24 feet; required the solidity.

Thus, $1·5 \times 1·5 = 2·25$; $1·25 \times 1·25 = 1·5625 + 2·25 + 1·875 = 5·6875 \times (24 \div 3 = 8) = 45·5$ feet. Ans.

2. What is the number of cubic feet in a stick of hewn timber whose ends are 30 inches by 27 inches, and 24 inches by 18 inches, the length being 24 feet? Ans. 102 feet.

Having the area of the end of a square piece of timber, to find the length which must be cut off to obtain a given solidity.

RULE 1.—Reduce the given solidity to cubic inches.

2. Divide the number of cubic inches by the area of the end expressed in inches, and the quotient will be the length in inches.

1. A piece of timber is 10 inches square; how much must be cut off to make a solid foot?

Thus, $10 \times 10 = 100$ square inches.

Then $1728 \div 100 = 17·28$ inches. Ans.

2. A piece of timber is 9 inches broad and 6 inches deep; how much in length will make 3 solid feet? Ans. 8 ft.

ROUND TIMBER.

To find the solidity of round timber.

RULE 1.—Take the girt or circumference, and then divide it by 5.

2. Multiply the square of one-fifth of the girt by twice the length, and the product will be the solidity, nearly.

1. A piece of timber is 9·75 feet in length, and the girt is 13 feet; what is its solidity?

Thus, 13 ÷ 5 = 2·6, the fifth of the girt.

Then $2·6^2$ = 6·76; and 9·75 × 2 = 19·5.

And 6·75 × 19·5 = 131·82 cubic feet. Ans.

2. Required the content of a piece of timber, its length being 9 feet 6 inches, and girt 14 feet. Ans. 148·96 cubic feet.

3. What is the solid content of a round piece of timber whose girt is 28·75 inches, and length 16 feet? Ans. 7·3472 ft.

4. What is the solid content of a piece of timber, its length being 16 feet, and girt 8¼ feet?

When the log tapers regularly from one end to the other.

RULE 1.—Gird the timber at as many points as may be necessary, and divide the sum of the girts by their number, for the mean girt; of which take one-fifth, and proceed as before.

1. If a tree girt 14 feet at the thicker end, and 2 feet at the smaller end, and 24 feet in length, how many solid feet will it contain? Ans. 122·88.

RULE 2.—To the product of the diameter add one-third of the square of their difference; multiply the sum by ·7854, and the product multiplied by the length will give the solidity; or to the product of the circumferences add one-third of the square of their differences; multiply the sum by ·07958, and the product multiplied by the length will give the solidity.

2. What is the solidity of a log, the diameter of the greater end being 4 feet, that of the less 2 feet, and the length 27 feet?

Thus, $4 \times 2 = 8;\ 2 \times 2 = 4;\ 8 - 4 \div 3 = \frac{4}{3} + 8 = 9\frac{1}{3}$.

Then $·7854 \times 9\frac{1}{3} = 7·3304 \times 27 = 197·9208$ solid feet. Ans.

3. If a log girt 6 feet at the thicker end, and 3 feet at the smaller end, required the solidity when the length is 24 feet. Ans. 40·1083 feet.

4. A tree girts at five different places as follows:—in the first, 9·43 feet; in the second, 7·92 feet; in the third, 6·15 feet; in the fourth, 4·74 feet; and in the fifth, 3·16 feet, and the length of the tree is 17·25 feet; what is its solidity?

Ans. 54·42499 cubic feet. Rule 1.

To find the number of cord feet in a log.

RULE.—Multiply the square of the diameter in inches by the length, and divide by 144.

One-third of the girt or circumference will be the diameter nearly.

1. How many tons of hewn or split timber are there in a log 30 feet in length, and 2 feet in diameter?

Thus, 2 ft. = 24 in. × 24 = 576 × 30 = 17280 ÷ 144 = 120 cord feet.

Then 120 ÷ 50 = $2\frac{2}{5}$ tons.

2. In ten logs, each 12 feet in length, and whose diameters are each 18 inches, how many cords? Ans. $2\frac{7}{64}$ cords.

Saw-Logs.

Rule.—From the diameter of the log subtract the thickness of the slabs, the remainder will be the width; from this deduct the waste of the saw; the remainder will be the number of boards.

Multiply the number of boards by the width, and that product by the length of the log; the last product divided by 12, will give the number of feet. Or, from the diameter of the log in inches, subtract 4 for the slabs. Then multiply the remainder by half itself, and the product by the length of the log in feet, and divide the result by 8; the quotient will be the number of square feet.

1. What is the number of feet of boards which can be cut from a log 12 feet in length, and 2 feet in diameter?

Thus, diameter 24 in. — 4 for slabs = 20 rem., half rem. = 10; 20 × 10 = 200 × 12 length of log = 2400 ÷ 8 = 300, number of feet.

2. How many feet can be cut from a log 28 inches in diameter and 14 feet long? Ans. 504 feet.

3. From a log 24 feet in length and 30 inches in diameter, how many feet of boards can be sawn, allowing 3 inches for slabs, and $\frac{1}{8}$ for waste of the saw? Ans. 1166·4 feet.

4. How many feet can be cut from a log 20 inches in diameter and 16 feet long? Ans. 256 feet.

5. How many feet can be cut from a log 12 inches in diameter and 12 feet long? Ans. 48 feet.

6. How many feet can be cut from a log 32 inches in diameter and 20 feet long?

Masons' Work.

Masons measure walls by the solid perch, which is $16\frac{1}{2}$ feet in length, 1 foot in depth, and $1\frac{1}{2}$ in breadth; this is equal to 24·75 solid feet, for $16\frac{1}{2}$ ÷ 2 = $8\frac{1}{4}$ + $16\frac{1}{2}$ = 24·75 solid feet;

whereas, if the breadth of the wall was but 1 foot, there would be but just $16\frac{1}{2}$ solid feet; hence the reason why 24·75 feet are called a perch, for this is the actual number of solid feet in a wall $16\frac{1}{2}$ feet in length, 1 foot in depth, and $1\frac{1}{2}$ in breadth. If walls are more than the above thickness, the excess is charged accordingly. Solid measure is generally used for materials, and the superficial for workmanship.

RULE.—In solid measure, multiply the length, breadth, and thickness continually together. And in superficial measure, the length and breadth of every part of the projection must be taken.

1. Required the solid content of a wall whose length is 48·5 feet, its height 10·75, and thickness 2 feet.

Thus, $48{\cdot}5 \times 10{\cdot}75 \times 2 = 1042{\cdot}75$ solid feet. Ans.

2. What is the solid content of a wall whose length is 60·75 feet, its height 10·25 feet, and thickness 2·5 feet?

Ans. 1556·71875 feet.

3. What is a marble slab worth whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 80 cents per foot?

Thus, 5 ft. 7 in. $= 5\frac{7}{12}$ ft. $= \frac{67}{12}$; and 1 ft. 10 in. $= 1\frac{5}{6}$ feet $= \frac{11}{6}$ ft.; $\frac{67}{12} \times \frac{11}{6} = \frac{737}{72} =$ content in feet; $\frac{737}{72} \times 80$ cents $= \frac{58960}{72}$ cents $= \frac{7370}{9} =$ D. 8·18·$8\frac{8}{9}$ m. Ans.

4. How many solid perches of stone are contained in a cellar wall, the length being 45·5 feet on one side, and the breadth within 24 feet at each end, $6\frac{3}{4}$ feet high, and 2 feet thick?

Ans. 118·72 perches.

5. Required the cost of making a stone wall under a building whose length is 42 feet, breadth on the outside 26 feet; the height of the wall being 6·5 feet, and 2 feet in thickness, at 40 cents per solid perch? Ans. D. 40·339 +

6. Required the solid content of a wall 82 feet 9 inches long, 20 feet 3 inches high, and 2 feet 3 inches thick.

Ans. 3770·2968 feet.

7. What is the solid content of a wall, the length of which is 24 feet 3 inches, height 10 feet 9 inches, and thickness 2 feet?

Ans. 521·375 feet.

CISTERNS.

A cistern is a large reservoir constructed to hold water, and is generally made of brick or masonry.

To find the number of hogsheads which a cistern of given dimensions will contain.

RULE.—Find the solid content of the cistern in cubic inches. Divide the content so found by $(231 \times 83) = 14553$, and the quotient will be the number of hogsheads.

1. The diameter of a cistern is 6 feet 6 inches, and height 10 feet; how many hogsheads will it contain?

Thus, 6 ft. 6 in. = 78 inches; $10 \times 12 = 120$ inches; $78^2 = 6084 \times 120$ in. $= 730080 \times \cdot 7854 = 573404\cdot 8320 \div 14553 = \cdot 39\cdot 40$ hogsheads.

2. The diameter of a cistern is 14 feet, and height 18 feet; how many hogsheads will it contain?

The diameter of a cistern being given, to find the height, so that the cistern shall contain a given number of hogsheads.

RULE.—Reduce the content to cubic inches; reduce the diameter to inches, and then multiply its square by ·7854. Divide the content by the last result, and the quotient will be the height in inches.

1. The diameter of a cistern is 8 feet; what must be its height that it may contain 150 hogsheads?

Ans. 25 ft. 1 in. nearly.

NOTE.—If the cistern is *square*, find the content in cubical inches, which bring to gallons or hogsheads.

BRICKLAYERS' WORK.

There are several methods by which bricklayers compute their work, generally at the rate of one and a half bricks in thickness; and if the wall be more or less than this standard, it can be reduced to it if required:—the rod square of 16½ feet, or 272¼ square feet; and that of 18 feet, or 324 square feet, and that which is 16½ feet long and 1 foot high, &c. It is the general practice in this country, at the present time, to calculate bricks by the 1000, but this must depend upon the size of the bricks. All windows, doors, &c. are to be deducted out of the contract of the walls in which they are placed; but the deduction is made only with regard to materials; for the value of their workmanship is added to the bill at the rate agreed upon.

The size of bricks in common use is 8 inches in length, 4 inches in width, and thickness 2 inches; or 64 cubic inches in 1 brick, and 27 bricks in 1 cubic foot. In estimating the number of bricks for a wall or house, it is necessary to know the *size* of the bricks to be used in the construction. A brick 9 inches long, width 4½ inches, thickness 2¼ inches, will contain 91⅛ cubic inches, and nearly 19 bricks in a cubic foot.

To find the number of bricks required to build a wall of given dimensions.

RULE 1.—Find the content of the wall in cubic feet.

2. Multiply the number of cubic feet by the number of bricks in a cubic foot, and the result will be the number of bricks required.

1. How many bricks, of 8 inches in length, will be required to build a wall 30 feet long, a brick and a half thick, and 15 feet high? Ans. 12150.

2. How many bricks, of the usual size, will be required to build a wall 50 feet long, 2 bricks thick, and 36 feet in height? Ans. 64800.

Thus, $36 \times 50 = 1800$ feet; 2 bricks in thickness equal 16 inches, or 1⅓ feet; $1800 \div 3 = 600 + 1800 = 2400$, number of cubical feet in the wall; then 2400×27 number of bricks in a cubical foot $= 64800$. Ans.

BRICK WORK.

RULE.—Multiply the total number of feet in the wall by the number of half bricks in the thickness of it, and divide the product by 3, which will give the standard measure. Then divide by 272¼, or by 324, as the case may require, and the quotient will be the number of rods required; the ¼ may be omitted.

1. If a wall measure 125 feet in length, 12 feet high, and 2½ bricks thick, how many rods does it contain?

Thus, $125 \times 12 \times 5 \div 3 = 2500 \div 272 = 9\frac{1}{68}$ rods. Ans.

2. How many rods are contained in a wall whose length is 67 feet, and height 32 feet 6 inches, the wall being 3 bricks thick? Ans. 16·01 rods.

3. What is the cost of a wall 60 feet long, 17½ feet high, and 2½ bricks thick, at D. 7·50 per thousand?

Ans. D. 354·37½.

CARPENTERS' AND JOINERS' WORK.

By carpenters' and joiners' work we are to understand all kinds of wood work used in buildings, and is measured by the square of 100 feet. In some cases the framing is measured by the square foot, in others by long measure.

Joiners' work is generally measured by the yard or foot; in measuring doors, window-cases, &c., each piece is measured by itself. A roof is said to have a *true pitch* when the length of the rafters is three-fourths the breadth of the building, as the rafters are then at nearly right angles. In all kinds of artificers' work the calculations must be made in accordance with the conditions of the contract, which in many cases differ from any *written* rule.

1. How many squares, of 100 square feet each, in a floor 48 feet 6 inches long, and 24 feet 3 inches broad?

Ans. 11 and 76⅛ square feet.

2. How many squares are there in a partition 91 feet 9 inches long, and 11 feet 3 inches high? Ans. 10 and 32 sq. feet.

3. What is the expense of flooring a building 45 feet 6 inches long, 26 feet 9 inches wide, two stories high, at D. 1·45 per square of 100 feet? Ans. D. 35·296.

4. If a floor is 90 feet long and 40 feet broad, how many squares will it contain; and what will be the cost of the boards at 1½ cents per square foot? Ans. 36 squares, and D. 54·00.

5. What will it cost to enclose a beam as high as the eaves, which is 60 by 45 feet, and 20 feet high to the eaves, at D. 1·85 per square of 100 feet; and what will be the cost of the boards at D. 4·75 per M?

6. What will it cost to lay the shingles on a roof, each side being 18 feet by 35 feet, at 75 cents per square yard?

SLATERS' AND TILERS' WORK.

In slating and tiling, the content of a roof is found by multiplying the length of a ridge by the girt from eave to eave; in slating, allowances must be made for the double row at the bottom. In taking the girt, the line is made to ply over the lowest row of slates, and returned the under side till it meets with the wall or eaves-board; but in tiling, the line is stretched down only to the lowest part, without returning it up again. For hips, valleys, gutters, &c., double measure is generally allowed, but no deductions are made for chimneys, &c. In all works of this

kind, the content is computed by the square yard, or the square of 100 feet.

1. The length of a slated roof is 48·5 feet, and its girt 36·25 feet; required the content.

Thus, 36·25 × 48·5 feet = 1758·125 square feet. Ans.

2. What will the tiling of a barn cost at D. 1·25 per square yard, the length being 42 feet, and depth 30 feet 6 inches on the flat, the eaves projecting 15 inches, the roof being a true pitch?

Ans. D. 281·458.

3. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches; required the content.

Ans. 1566·9375 square feet.

4. What is the content of a slated roof, the length being 50 feet 9 inches, and the whole girt 35 feet 6 inches?

5. Required the content of a tiled roof, the length being 65 feet, and the whole girt 45 feet.

Plasterers' Work.

Plasterers' work is of two kinds, namely, ceiling, which is plastering on laths; and rendering, which is plastering on walls: these are measured separately. The contents are estimated either by the square foot, the square yard, or by the square of 100 feet, except cornice, mouldings, &c., which is measured by running or linear measure. Deduction is made for doors and windows. Whitening and colouring are measured in the same manner as plastering.

1. If the partitions between rooms are 141·5 feet about, and 11·25 feet high, how many yards do they contain?

Thus, 141·5 × 11·25 = 1591·875 ÷ 9 = 176·875 yds. Ans.

2. How many square yards are contained in a ceiling 43 feet 3 inches long, and 25 feet 6 inches broad?

Ans. 122½ nearly.

3. In a room 25 feet 6 inches long, 16 feet wide, and 9 feet 3 inches high, what will the plastering of the ceiling and walls come to, at 25 cents per yard? Ans. D. 32·659.

4. If a ceiling be 64·75 feet long, and 24·5 broad; how many square yards does it contain? Ans. 176·263⅝ yards.

5. The four walls of a church are each 84 feet in length and 32 in height; required the number of square yards in the four walls, and the cost of plastering at 8 cents per square yard.

6. In a dining saloon the length is 75·5 feet, width 36 feet, height to ceiling 18 feet; required the number of square yards

in the four walls, and also in the ceiling, and the cost of the whole at $12\frac{1}{2}$ cents per square yard.

PAINTERS' WORK.

Painters' work is computed in square yards; every part is measured where the colour lies, and the measuring line is carried into all the mouldings and cornices.

Windows are generally done by the piece; it is usual to allow double measure for carved mouldings, &c.

1. How many yards of painting in a room which is 65 feet 6 inches in perimeter, and 12 feet 4 inches in height?

Ans. $89\frac{41}{54}$ square yards.

2. If a room be painted, whose height is 12 feet 4 inches, and its compass 111 feet 3 inches, how many yards does it contain, and what will it cost at 18 cents per yard?

Thus, 111 ft. 3 in. × 12 ft. 4 in. = 1372 ft. 1 in.; and 1372 ft. 1 in. ÷ 9 = 152·45 yards × 18 = D. 27·44, amount.

3. The length of a room is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it, deducting a fireplace of 4 feet by 4 feet 4 inches, and two windows, each 6 feet by 3 feet 2 inches?

Ans. $73\frac{2}{27}$ square yards.

PAVERS' WORK.

Pavers' work is done by the square yard; and the content is found by multiplying the length and breadth together.

1. What is the cost of paving a sidewalk, the length of which is 35 feet 4 inches, and breadth 8 feet 3 inches, at 54 cents per square yard? Ans. D. 17·48·9.

2. How many yards are there in a sidewalk, the length of which is 396 feet, and breadth 12 feet 4 inches?

Ans. $542\frac{2}{3}$.

3. How many bricks will it require to pave a square yard, whose side is 90 feet, allowing 42 bricks to a square yard?

GLAZIERS' WORK.

Glaziers' take their dimensions either in feet, inches, and parts; or feet, tenths, and hundredths; they compute their work in square feet. It is the common practice to take the whole window-frame for glazing, and multiply its superficies by the number of panes.

1. What will the glazing of a triangular skylight come to at 20 cents per foot, the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches?

Thus, $12{\cdot}5 \times 3{\cdot}375 = 42{\cdot}1875$ feet $\times 20 =$ D. $8{\cdot}43\frac{3}{4}$. Ans.

2. In a court-house there are 16 windows, each 8 feet 6 inches by 4 feet 8 inches; required the number of square feet in the 16 windows, and cost of glazing at 15 cents per square foot.

Plumbers' Work.

Plumbers' work is estimated by the pound, or hundred weight of 112 pounds. Sheet lead used for gutters, &c., weighs from 6 to 12 pounds per square foot; leaden pipes vary in weight according to the diameter of their base and thickness. The following table shows the weight of a square foot of sheet lead according to its thickness; and weight of a yard of leaden pipe according to its diameter:

Thickness of lead.	Pounds to a square foot.	Bore of leaden pipe.	Pounds per yard.
Inch $\frac{1}{10}$	5·899	$0\frac{3}{4}$	10
$\frac{1}{9}$	6·554	1	12
$\frac{1}{8}$	7·373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8·427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9·831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11·797	2	24

1. Required the weight of a sheet of lead which is 20 feet 6 inches in length, and 7 feet 9 inches in breadth, and $8\frac{1}{4}$ pounds to the square foot.

Thus, $20{\cdot}5 \times 7{\cdot}75 = 158{\cdot}875 \times 8\frac{1}{4} = 1310{\cdot}719$ lbs. $= 11$ cwt. 2 qrs. $22\frac{3}{4}$ lbs. Ans.

2. What will be the cost of 130 yards of leaden pipe of an inch and a half base, at 8 cents per pound, allowing each yard to weigh 18 pounds? Ans. D. 187·20.

Bins of Grain.

It is the practice of farmers to make *bins* or boxes for the storage of their grain. It is frequently necessary to ascertain the quantity of grain a box of given dimensions will contain; or what size the box must be to contain a given quantity of grain.

Since a bushel contains 2150·4 cubic inches, it follows that a bushel contains one and a quarter cubic feet nearly; then by having any number of bushels, you can find the corresponding number of cubic feet by increasing the number of bushels *one-fourth* itself, and the result will be the number of cubic feet.

1. A bin contains 372 bushels; required the number of cubic feet.

Thus, $372 \div 4 = 93 + 372 = 465$ cubic feet.

2. In a bin containing 400 bushels, how many cubic feet?

Ans. 500.

To find the number of bushels which a bin of a given size will hold.

Rule.—Find the content of the bin in cubic feet; then diminish the content by $\frac{1}{5}$, and the result will be the content in bushels.

3. In a bin 8 feet long, 4 feet wide, and 5 feet high, how many bushels may be held?

Thus, $8 \times 4 \times 5 = 160 \div 5 = 32$; $160 - 32 = 128$ bushels. Answer.

In the following question increase the dimensions $\frac{1}{4}$.

4. What must be the height of a bin that will contain 600 bushels, its length being 8 feet, and breadth 4 feet?

Thus, $600 \div 4 = 150 + 600 = 750$ cubic feet.

And $8 \times 4 = 32$, the product of the given dimensions.

Then $750 \div 32 = 23·44$ feet, the height of the bin.

Vaulted and Arched Roofs.

Arched roofs are either vaults, domes, saloons, or groins.

Vaulted roofs are formed by arches springing from the opposite wall, and meeting in a base at the top.

Domes are made by arches springing from a circular or polygonal base, and meeting in a point at the top.

Groins are formed by the intersection of vaults with each other.

Circular roofs are those whose arch is some part of the circumference of a circle.

Elliptical roofs are those whose arch is some part of the circumference of an ellipsis.

Gothic roofs are those which are formed by two circular arc

that meet in a point directly over the middle of the breadth or span of the arch.

To find the solid content of a circular, elliptical, or Gothic vaulted roof.

RULE.—Find the area of one end, and multiply it by the length of the roof; the product will be the solidity required.

1. Required the solidity of an elliptic vault whose span is 40 feet, height 12 feet, and length 80 feet.

Thus, $12 \times 40 \times \cdot 7854 = 376\cdot 992$ = area of one end.

And $376\cdot 992 \times 80 = 30159\cdot 36$ feet, solidity required.

2. What is the solid content of a semi-circular vault whose span is 40 feet, and its length 120 feet?

Thus, $40 \times 40 = 1600 \times \cdot 7854 = 1256\cdot 64 \div 2 = 628\cdot 32$.

Then $628\cdot 32 \times 120 = 75398\cdot 40$. Ans.

3. What is the solidity of a semi-circular vault whose span is 40 feet, and length 124 feet? Ans. 77911·68.

To find the concave or convex surface of a circular, elliptical, or Gothic vaulted roof.

RULE.—Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

1. What is the surface of a vaulted roof, the length of the arch being 35 feet, and the length of the vault 140 feet?

Thus, $35 \times 140 = 4900$ square feet, the surface required.

2. Required the surface of a vaulted roof, the length of the arch 40 feet 6 inches, and the length of the vault 79 feet.

Ans. 3199·5 feet.

To find the solid content of a dome, its height and the dimensions of its base being known.

RULE.—Multiply the area of the base by two-thirds of the height, and the product will be the solidity.

1. What is the solid content of a spherical dome, the diameter of whose circular base is 60 feet, and height 30 feet?

Thus, $60 \times 60 = 3600 \times \cdot 7854 = 2827\cdot 44 \times 20$ ($\frac{2}{3}$ of 30) = 56548·8. Ans.

2. What is the solid content of an octagonal dome, each side of its base being 20 feet, and the height 21 feet?

Ans. 27039·1912 feet.

To find the superficial content of a spherical dome.

RULE.—Multiply the area of the base by 2, and the product will be the superficial content required.

1. Required the superficial content of an hexagonal spherical dome, each side of the base being 20 feet.

Thus, $20 \times 20 = 400 \times 2{\cdot}598076 =$ (area of a hexagon whose side is 1,) $1039{\cdot}2304 =$ area of base $\times 2 = 2078{\cdot}4608$. Answer.

2. What will be the expense of painting an hexagonal spherical dome, each side of whose base is 20 feet, at 15 cents per square yard? Ans. D. 34·641.

To find the solid content of the vacuity formed by a groin or arch, either circular or elliptical.

RULE.—Multiply the area of the base by the height, and the product by ·904, and it will give the solidity required.

1. What is the solid content of the vacuity formed by a circular groin, one side of its square base being 12 feet, and length 6 feet?

Thus, $12^2 \times 6 \times {\cdot}904 = 781{\cdot}056 =$ solidity required.

2. What is the solid content of the vacuity formed by an elliptical groin, one side of its square base being 20 feet, and height 6 feet? Ans. 2169·6 feet.

To find the concave superficies of a circular groin.

RULE.—Multiply the area of the base by 1·1416, and the product will be the superficies required.

1. What is the curve superficies of a circular groin arch, one side of its square being 12 feet?

Thus, $12 \times 12 \times 1{\cdot}1416 = 164{\cdot}3904$, superficies required.

2. What is the concave superficies of a circular groin arch, one side of its square being 9 feet? Ans. 92·4696.

To find the superficies of a saloon.

RULE.—Find its breadth by applying a line close to it across the surface; find also its length by measuring along the middle of it, quite round the room; then multiply these two dimensions together for the superficial content.

1. The girt across the face of the saloon is 4 feet, and its mean compass 99 feet; what is the superficial content?

Thus, $99 \times 4 = 396$ feet, the superficial content.

2. The girt across the face of a saloon is 24 feet, and the mean compass 196 feet; required its surface.

3. The girt across the face of the saloon is 18 feet, its mean compass 184 feet; required its surface.

4. The girt across the face of the saloon is 28 feet, its mean compass 224 feet; required its surface.

Specific Gravity.

The specific gravity of bodies are their relative weights contained under the same given magnitude, as a cubic foot, a cubic inch, &c.; and are expressed by the numbers annexed to their names in the following table, as found by actual experiments, and are calculated to correspond with a cubic foot of each avoirdupois.

It has been ascertained that a cubic foot of rain water weighs 62½ pounds, or 1000 ounces avoirdupois; and a cubic foot containing 1728 cubic inches, it follows that a cubic inch weighs ·03616898148 of a pound; hence by multiplying the specific gravity of a body by the above number, the product will be the weight of a cubic inch of that body in pounds avoirdupois, which being multiplied by 175, and the product divided by 144, the quotient will be the weight of a cubic inch in pounds troy—144 pounds avoirdupois being equal to 175 pounds troy.

Table of Specific Gravities.

Names of Materials.	Specific Gravity.	Weight of 1 cubic foot in lbs.
Platina, hammered,	20·337	1271·81
Very fine gold	19·639	
Platina	19·500	1218·76
Pure cast gold	19·258	1204·42
Standard gold	18·888	
Guinea gold	17·793	
Moidore gold	17·144	
Mercury, at 32°	13·619	851·18
Cast lead	11·344	709·00
Fine silver	10·474	654·91
Standard silver	10·312	644·5
Rose copper	9·000	

Names of Materials.	Specific Gravity.	Weight of 1 cubic foot in lbs.
Cast copper - - - - - -	8·788	549·25
Cast brass - - - - - -	8·306	524·75
Hard steel - - - - - -	7·816	488·50
Bar iron - - - - - -	7·788	485·57
Cast tin - - - - - -	7·291	455·68
Cast iron - - - - - -	7·271	454·43
Cast zinc - - - - - -	7·190	449·37
Lead ore - - - - - -	6·800	
Copper ore - - - - - -	3·775	
Limestone - - - - - -	3·179	198·68
Diamond - - - - - -	3·400	
Crystal glass - - - - -	3·150	
White glass - - - - - -	2·892	
Chalk - - - - - -	2·784	174·00
Marble - - - - - -	2·742	171·38
Alabaster - - - - - -	2·730	
Pearl - - - - - -	2·684	
Flint - - - - - -	2·582	
Paving stone - - - - - -	2·578	
Cornelion - - - - - -	2·568	
Green glass - - - - - -	2·642	
Common stone - - - - -	2·520	157·50
Sulphur - - - - - -	2·033	127·06
Brick - - - - - -	2·000	125·00
Ivory - - - - - -	1·822	
Nitre - - - - - -	1·900	
Gunpowder - - - - - -	1·714	
Bone of an ox - - - - -	1·659	
Honey - - - - - -	1·456	
Lignum vitæ - - - - -	1·333	83·31
Ebony - - - - - -	1·331	83·18
Oak - - - - - -	1·170	73·12
Human blood - - - - -	1·054	
Amber - - - - - -	1·078	
Milk - - - - - -	1·030	
Sea water - - - - - -	1·028	
Distilled water - - - - -	1·000	
Liquid turpentine - - - -	·991	
Burgundy wine - - - - -	·991	

Names of Materials.	Specific Gravity.	Weight of 1 cubic foot in lbs.
Camphor	·989	
Oak, English,	·970	60·62
Bees' wax	·965	
Tallow	·945	
Olive oil	·915	
Logwood	·913	57·06
Box, French,	·912	57·00
Wax	·897	
Oak, Canadian,	·872	54·50
Alder	·800	50·00
Apple tree	·793	49·56
Ash	·760	47·50
Maple	·750	46·87
Cherry tree	·715	44·68
Beech	·696	43·50
Elder	·695	43·44
Walnut	·671	41·94
Pear	·661	41·31
Pitch pine	·660	41·25
Cedar	·596	37·25
Mahogany	·560	35·00
Elm	·556	34·75
Larch	·544	34·00
Poplar	·383	23·94
Cork	·240	15·00
Air, at the earth's surface,	·0012	
Inflammable air	·00012	

Note.—All bodies expand with heat and contract with cold, but some more and some less than others; consequently the specific gravity of bodies is not precisely the same in summer as in winter.

The specific gravity of a body, and its weight being given, to find its solidity.

Rule.—As the tabular specific gravity of the body is to its weight in avoirdupois ounces, so is one cubic foot, or 1728 cubic inches, to its content in feet or inches respectively.

1. Required the content of an irregular block of common stone, which weighs 1 cwt. or 112 pounds.

Thus, 2520 oz. : 1722 oz. : : 1728 cubic in. : $1228\frac{4}{5}$ cubic inches. Ans.

2. How many cubic inches of gunpowder are there in one pound? Ans. 14·91.

3. How many cubic feet are there in a ton of dry oak, its specific gravity being ·925? Ans. $38\frac{138}{185}$ feet.

The linear dimensions or magnitude of a body being given, and its specific gravity, to find its weight.

RULE.—As one cubic foot, or 1728 cubic inches, is to the solidity of the body, so is the tabular specific gravity of the body to the weight in avoirdupois ounces.

1. Required the weight of a block of marble whose length is 63 feet, and its breadth and thickness each 12 feet, and specific gravity 27 ounces.

Thus, $12^2 \times 60 = 9072$ solid feet.

Then 1 : 9072 : : 2700 : 244944 oz. $= 694\frac{11}{160}$ tons.

2. What is the weight of a block of dry oak, which measures 10 feet in length, and 3 feet in breadth, and 2·5 feet deep; specific gravity ·925? Ans. $4335\frac{15}{16}$ lbs.

To find the specific gravity of a body, when the body is heavier than water.

RULE.—Weigh it both in and out of water, and the difference will be the weight lost in the water; then as the weight lost in the water is to the whole weight, so is the specific gravity of water to the specific gravity of the body.

1. A piece of platina weighed 83·1886 pounds out of water, and in water only 79·5717 pounds; required its specific gravity, that of water being 1000.

Thus, 83·1886 — 79·5717 = 3·6169 pounds, which is the weight lost in water.

Then 3·6169 : 83·1886 : : 1000 : 23000, the specific gravity, or the weight of a cubic foot of metal in ounces.

2. A piece of stone weighed 10 pounds in the air, but in water only 6·75; what is the specific gravity?

Having the magnitude and weight of any body given, to find the specific gravity.

RULE.—Divide the weight by the magnitude, and the quotient will be the specific gravity.

1. A piece of marble contains 8 cubic feet, and weighs 1353·5 pounds, or 21656 ounces; required the specific gravity.
Ans. 2707.

To find the quantity of pressure against the sluice or bank which contains water.

RULE.—Multiply the area of the sluice under water by the depth of the centre of gravity (which is equal to half the depth of water) in feet, and that product again by 62·5, the number of pounds avoirdupois in a cubic foot of fresh water, or by 64·4, the avoirdupois weight of a cubic foot of salt water, and the product will be the number of pounds required.

1. The length of a sluice or floom being 30 feet, the width at the bottom 4 feet, and the depth of water 4 feet; what is the pressure of water against the sluice?

Thus, $30 \times 4 = 120$ feet, area of bottom; and 120×2 (the depth of the centre of gravity) $= 240$ cubic feet; and $240 \times 62\frac{1}{2} = 15000$ lb. $=$ 6 tons, 13 cwt. 3 qrs. 20 lb. Ans.

GAUGING, OR MEASURING CASKS.

There are four different forms of casks, which are frustums of different kinds of solids, named from the greater or less curvature of their sides.

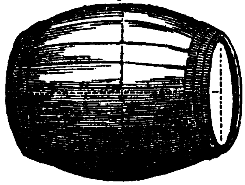
1. The middle frustum of a prolate spheroid.
2. The middle frustum of a parabolic spindle.
3. The two equal frustums of a paraboloid.
4. The two equal frustums of a cone.

To find the content of a cask of the first variety.

RULE.—To the square of the head diameter add double the square of the bung diameter, and multiply the sum by the length of the cask; then let the product be multiplied by $\cdot 0009\frac{1}{4}$ for ale gallons, and multiplied by $\cdot 0011\frac{1}{3}$ for wine gallons.

1. Required the content of a spherical cask whose length is 40 inches, and head and bung diameters 24 and 32 inches.

Fig. 1.



Thus, $24^2 = 576 : 32^2 = 1024 \times 2 = 2048 + 576 = 2624 \times 40 = 104960$.

Then $104960 \times \cdot0009\frac{1}{4} = 97\cdot0880$ ale gallons.

And $104960 \times \cdot0011\frac{1}{3} = 118\cdot9547$ wine gallons.

2. What is the content of a spheroidal cask whose length is 45 inches, bung diameter 34 inches, and head diameters each 25 inches, in ale gallons?

Ans. 122·716 ale gallons.

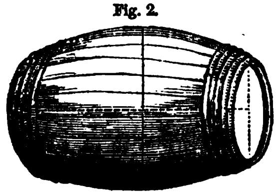
3. A spheroidal cask is 42 inches long, diameter at the bung 32 inches, and the head diameters each 23 inches; what is the content in wine gallons? Ans. 122·7147 wine gallons.

To find the content of a cask of the second form.

Rule.—To the square of the head diameter add double the square of the bung diameter, and from the sum take $\frac{2}{5}$ of the square of the difference of the diameters; then multiply the remainder by the length, and the product again by $\cdot0009\frac{1}{4}$ for ale gallons, or by $\cdot0011\frac{1}{3}$ for wine gallons.

1. The length being 40 inches, and diameters 24 and 32 inches; required the content.

Fig. 2.



Thus, $32 - 24 = 8 \times 8 = 64 \times 4 = 25\cdot6 : 32^2 = 1024 \times 2 = 2048 : (32 - 24)^2 \times \frac{2}{5} = 576 + 2048 = 2624 - 25\cdot6 = 2598\cdot4$.

Then $2598\cdot4 \times 40 = 10393\cdot6 \times 0009\frac{1}{4} = 96\cdot1408$ ale gallons.

And $103936 \times \cdot0011\frac{1}{3} = 117\cdot9741$ wine gallons.

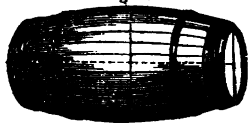
2. What is the content in wine gallons, of a cask whose length is 40 inches, bung diameter 31 inches, and head diameter 24 inches? Ans. 122·7143 gallons.

To find the solidity of a cask of the third variety.

Rule.—To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and the

product again by ·0014, for ale gallons, or by ·0017, for wine gallons.

Fig. 3.



1. Required the content of a cask of the third form, whose length is 40, and the diameters 24 and 32 inches.

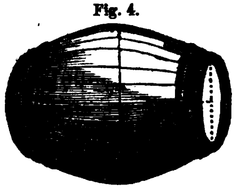
Thus, $24^2 = 576 : 32^2 = 1024 + 576 = 1600 \times 40 = 6400 \times ·0014 = 89·6$ ale gallons; $6400 \times ·0017 = 108·8$ wine gallons.

2. Required the content in ale gallons of a cask of the third form, whose length is 50 inches, bung diameter 30 inches, and head diameter 20 inches. Ans. 91 ale gallons.

To find the content of a cask of the fourth variety.

RULE.—Add the square of the difference of the diameters to three times the square of their sum; then multiply the sum by the length, and the product again by ·00023⅕ for ale gallons, or by ·00028⅓ for wine gallons.

Fig. 4.



1. Required the content when the length is 40, and the diameters 24 and 32 inches.

Thus, $32 + 24 = 56 \times 56 = 3136 \times 3 = 9408 : 8 \times 8 = 64 + 9408 = 9472 \times 40 = 378880 \times ·00023⅕ = 87·90016$ ale gallons.

And $378880 \times ·00028⅓ = 107·34983$ wine gallons.

2. The length of a cask of the fourth form is 50 inches, the bung diameter 30 inches, and head diameter 21 inches; required the number of ale gallons. Ans. 91·49 + ale gallons.

3. The head diameters of a cask of the fourth form are 18 inches, the bung diameter 30 inches, and the length 50 inches; required the content in wine gallons. Ans. 99·96.

THE ULLAGE OF CASKS.

The ullage of a cask is what it contains when only partly filled. The cask is considered either standing on the end, with its axis perpendicular to the horizon, or as lying down on its side with the axis parallel to the horizon.

To ullage a standing cask.

RULE.—Add all together, the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two; then multiply the sum by the length between the surface and nearest end, and the product again by $\cdot0004\frac{2}{3}$ for ale gallons, or by $\cdot0005\frac{2}{3}$ for wine gallons in the less part of the cask, whether empty or filled.

1. The three diameters being 24, 27, and 29 inches; required the ullage for 10 wet inches.

Thus, $24 \times 24 = 576 : 29^2 = 841 : 54^2 = 2916 + 841 + 576 = 4333 \times 10 = 43330 \times \cdot0004\frac{2}{3} = 20\cdot2205$ ale gallons; $43330 \times \cdot0005\frac{2}{3} = 24\cdot5535$ wine gallons.

2. If the diameter at the surface of the liquor in a standing cask be 32 inches, the diameter of the nearest end 24 inches, the middle diameter 29 inches, and the distance between the surface of the liquor and the nearest end 12 inches; required the number of wine gallons in the cask. Ans. $33\frac{3}{4}$ gallons.

To ullage a lying cask.

RULE.—Divide the wet inches by the bung diameter; find the quotient in the column of versed sines, in the table of circular segments, take out its corresponding segment; then multiply this segment by the whole content of the cask, and the product again by $1\frac{1}{4}$ for the ullage required, nearly.

Thus, $8\cdot00 \div 32 = \cdot25$, whose tabular segment is $\cdot153546$.

Then $\cdot153546 \times 92 = 14\cdot126232 \times 1\frac{1}{4} = 17\cdot65779$ gallons.

NOTE.—The table of versed sines is omitted in this volume.

To find the content of a cask by the mean diameter.

RULE.—Multiply the difference of the head and bung diameters by $\cdot68$ for the first variety, by $\cdot62$ for the second variety, by $\cdot55$ for the third, and by $\cdot5$ for the fourth; when the difference between the head and bung diameters is less than 6 inches; but when the difference between these exceeds 6 inches, multiply that difference by $\cdot7$ for the first variety, by $\cdot64$ for the second, by $\cdot57$ for the third, and by $\cdot52$ for the fourth. Add this product to the head diameter, and the sum will be a mean diameter. Square this mean diameter, and multiply the square by the length of the cask; this product, divided by $359\cdot05$ for ale gallons, and by $294\cdot1$ for wine gallons.

1. Required the content of a cask of the first variety, in ale gallons, whose length is 40, bung and head diameters 32 and 24 inches.

Thus, $32 - 24 = 8 \times \cdot 7 = 5 \cdot 6 + 24 = 29 \cdot 6$, the mean diameter.

Then $29 \cdot 6^2 \times 40 = 876 \cdot 16 \times 40 = 35046 \cdot 4$.

And $35046 \cdot 4 \div 359 \cdot 05 = 97 \cdot 6$ ale gallons.

2. Required the content of a cask of the second variety, in wine gallons, whose length is 20, bung and head diameter 16 and 12 inches. Ans. 14·25 wine gallons.

Ships' Tonnage.—Carpenters' Tonnage.

There are several methods by which to ascertain the measurement of ships, the following rule is probably the most correct.

To find the tonnage by carpenters' measure.

Rule.—For single-decked vessels, multiply the length and breadth at the main beam, and depth of the hold together, and divide the product by 95. For double-decked vessels, take half the breadth of the main beam for the depth of the hold, and work as above.

If the deck be bolted at any height above the main wale, it is customary to add half the difference to the former depth, for the depth used in calculating the tonnage.

1. What is the tonnage of a single-decked vessel whose length is 65 feet, breadth 20 feet 6 inches, and depth 8 feet 9 inches?

Thus, $65 \times 20 \cdot 5 \times 8 \cdot 75 = 11659 \cdot 375 \div 95 = 122 \cdot 73$ tons.

2. What is the tonnage of a double-decked vessel whose length is 75 feet 9 inches, and breadth of main beam 24 feet 6 inches?

Thus, $75 \cdot 75 \times 24 \cdot 5 \times 12 \cdot 25 = 22734 \cdot 46875 \div 95 = 239 \cdot 31$ tons.

3. What is the tonnage of a ship whose length is 97 feet, breadth 31 feet, depth 15·5 feet?

By another method.—Thus, $\frac{1}{2}$ the breadth $= 15 \cdot 5 \times 31$ breadth $= 480 \cdot 5 \times 97$ length $= 46608 \cdot 5 \div 94 = 495 \cdot 83$ tons.

4. Required the tonnage of a ship, of which the length is 75 feet, and the breadth 26 feet.

Thus, $26^2 \times 75 \div 188 = 270$ tons, *nearly*.

The content of Noah's ark is as follows, allowing the cubit to be 22 inches; length of keel 300 cubits, breadth of the midship

beam 50 cubits, depth in the hold 30 cubits; its burthen as a man of war, 27729 tons; as a merchant ship, 29188·6 tons.

Falling Bodies.

When a body descends freely by its own weight, the velocity is as the time, and space as the square of the time. The time and velocity then will be 1, 2, 3, &c.; the space passed through as 4, 9, 16, &c.; and the spaces for each time as 1, 3, 5, 7, 9, &c. A falling body will descend through $16\frac{1}{12}$ feet in the first second of time, its velocity increasing so, that in the next second it will fall $32\frac{1}{6}$ feet.

A Table, showing the time, space, and velocity.

Seconds from the beginning of descent.	Velocity acquired at the end of that time.	Squares.	Space fallen through in that time.	Spaces.	Space fallen through in the last second of time.
1	$32\frac{1}{6}$	1	$16\frac{1}{12}$	1	$16\frac{1}{12}$
2	$64\frac{1}{3}$	4	$64\frac{1}{3}$	3	$48\frac{1}{4}$
3	$96\frac{1}{2}$	9	$144\frac{3}{4}$	5	$80\frac{5}{12}$
4	$128\frac{2}{3}$	16	$257\frac{1}{3}$	7	$112\frac{7}{12}$
5	$160\frac{5}{6}$	25	$402\frac{1}{12}$	9	$144\frac{3}{4}$
6	193	36	579	11	$176\frac{11}{12}$
7	$225\frac{1}{6}$	49	$788\frac{1}{12}$	13	$209\frac{1}{12}$
8	$257\frac{1}{3}$	64	$1029\frac{1}{3}$	15	$241\frac{1}{4}$
9	$289\frac{1}{2}$	81	$1302\frac{3}{4}$	17	$273\frac{5}{12}$
10	$321\frac{2}{3}$	100	$1608\frac{1}{3}$	19	$305\frac{7}{12}$
11	$353\frac{5}{6}$	121	$1946\frac{1}{12}$	21	$337\frac{3}{4}$
12	386	144	2316	23	$369\frac{11}{12}$

To find the velocity a falling body will acquire in a given time.

Rule.—Multiply the time in seconds by $32\frac{1}{6}$, and it will give the velocity required in feet per second.

1. Required the velocity in 15 seconds.

Thus, $15 \times 32\frac{1}{6} = 482\frac{1}{2}$ feet. Ans.

2. Required the velocity in 10 seconds. Ans. $321\frac{2}{3}$ ft.

To find the velocity a body will acquire by falling from any given height.

Rule.—Multiply the space in feet by $64\frac{1}{3}$, and the square root of the product will be the velocity acquired in feet per

second; or, when the time is given, multiply the time in seconds by $32\frac{1}{6}$.

1. Required the velocity a ball has acquired in descending through $144\frac{3}{4}$ feet.

Thus, $\sqrt{144\frac{3}{4} \times 64\frac{1}{3}} = \sqrt{9312 \cdot 25} = 96 \cdot 5$ feet.

2. Required the velocity a ball has acquired in descending through $402\frac{1}{12}$ feet in 5 seconds. Ans. $160\frac{5}{6}$ feet.

To find the space through which a body will fall in any given time.

Rule.—Multiply the square of the time in seconds by $16\frac{1}{12}$, and it will give the space in feet.

1. Required the space fallen through in 6 seconds.

Thus, $6^2 = 36 \times 16\frac{1}{12} = 579$ feet. Ans.

2. A bullet being dropped from the top of a building was 5 seconds in reaching the ground; required the height.

Ans. $402\frac{1}{12}$ feet.

To find the time that a body will be in falling through a given space.

Rule.—Divide the space in feet by $16\frac{1}{12}$, and the square root of the quotient will give the required time in seconds.

1. Required the time a body will be in falling through 579 feet of space. ($579 \div 16\frac{1}{12} = \sqrt{36} = 6$ seconds.)

2. How long will a body be in falling through $402\frac{1}{12}$ feet of space? Ans. 5 seconds.

To find the space fallen through, the velocity being given.

Rule.—Divide the velocity by 8·02, and the square of the quotient will be the distance fallen through to acquire that velocity.

1. If the velocity of a cannon ball be 579 feet per second, from what height must a body fall to acquire the same velocity?

Thus, $579 \div 8 \cdot 02 = 72 \cdot 19^2 = 5211 \cdot 3961$ feet.

2. If the velocity of a cannon ball be $689\frac{18}{25}$ feet per second, from what height must a body fall to acquire the same velocity?

To find the time, the velocity per second being given.

Rule.—Divide the velocity by $32\frac{1}{6}$, and the quotient will be the time in seconds.

1. How long must a bullet be in falling to acquire a velocity of 772 feet per second? ($772 \div 32\frac{1}{6} = 24$ seconds.)

2. How long must a bullet be in falling to acquire a velocity of 386 feet per second? Ans. 12 seconds.

Ascending bodies are retarded in the same ratio that descending bodies are accelerated.

To find the space moved through by a body projected upward with a given velocity.

RULE.—Multiply the square of the time in seconds by $16\frac{1}{12}$, (see Table) the velocity of the projection in feet by the number of seconds the body is in motion, and the sum of these products will be the space in feet when projected downward; and the difference of the products will give the distance of the body from the point of projection, when projected upward.

1. If a rocket, projected upward, return to the earth in 12 seconds, how high did it ascend?

The rocket is half the time in ascending $12 \div 2 = 6$; $193 \times 6 = 1158$; $6^2 = 36 \times 16\frac{1}{12} = 579$; $1158 - 579 = 579$. Answer.

2. A bullet is dropped from the top of a building, and found to reach the ground in 1·75 seconds; required the height.

$1·75 = 7$, and $7 \times 7 = 49$ feet. Ans.

Or, $1·75^2 \times 16 = 49$ feet. Ans.

3. In what time will a ball dropped from the top of a steeple 484 feet high, come to the ground?

$\sqrt{484} = 22 \div 4 = 5\frac{1}{2}$ seconds. Ans.

To find the velocity per second with which a heavy body will begin to descend, at any distance from the earth's surface.

RULE.—As the square of the earth's semi-diameter is to 16 feet, so is the square of any other distance from the earth's centre inversely to the velocity with which it began to descend per second.

1. With what velocity per second will an iron ball begin to descend, if raised 3000 miles above the earth's surface?

As $4000 \times 4000 : 16 :: \overline{4000 + 3000 + 3000} : 5·22449$ ft. Answer.

2. How high must a ball be raised above the earth's surface to begin to descend with a velocity of 5·22449 feet per second?

Thus, $16 : 4000 \times 4000 :: 5·22449 : 49000000$.

And $\sqrt{49000000} = 7000$; and $7000 - 4000 = 3000$ miles. Answer.

To find the velocity of a falling stream of water per second, at the end of any given time, the perpendicular distance being given.

RULE.—The velocity required at the end of every period is equal to twice the mean velocity with which it passes during that period. Or 2d, Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

1. There is a sluice or flume, one end of which is 2·5 feet lower than the other; what is the velocity of the stream per second?

Thus, $2{\cdot}5 \times 64 = 160$; and $\sqrt{160} = 12{\cdot}649$ feet. Ans.

The weight of a body, and the space fallen through given, to find the force with which it will strike.

RULE.—Find the velocity by the preceding rule, and multiply it by the weight, which will produce the weight required.

1. The rammer used for driving the piles of a bridge weighed 2¼ tons or 4500 pounds, and fell through a space of 10 feet, with what force did it strike the pile?

Thus, $\sqrt{10 \times 64} = 25{\cdot}3 =$ velocity; and $25{\cdot}3 \times 4500 = 113850$ pounds. Ans.

Then reverse the question and rule; if the aforementioned rammer weighed 4500 pounds, and struck with a force of 113850 pounds, from what height did it fall?

Thus, $113850 \div 4500 = 25{\cdot}3$; and $\overline{25{\cdot}3 \times 25{\cdot}3} \div 64 = 10$ feet. Ans.

To ascertain with what momentum or force a fluid moving with a given velocity strikes upon a fixed obstacle.

RULE.—Divide the square of the velocity by 64, and the quotient will be the height required. Multiply the height by 62½ pounds avoirdupois for clear water, and by 63 for unclean water, and by 64 for salt water.

1. Admit a stream of clear water to move at the rate of 5 feet per second, and to meet with a fixed obstacle (or bulk head) 15 feet wide and 4 feet high, what is the momentary pressure of the stream?

Thus, $5 \times 5 \div 64 = \frac{25}{64}$; and $25 \div 64 = \cdot 39$ of a foot, for the perpendicular fall of the water.

Then $62 \cdot 5 \times \cdot 39 = 24 \cdot 375$ pounds, the pressure upon each square foot, which multiplied by 60, (the number of square feet in the obstacle,) gives $1462 \cdot 5$ pounds, going with the given velocity of 5 feet per second; therefore $1462 \cdot 5 \times 5 = 7312 \cdot 5$ pounds. Ans.

The quantity of water discharged from a hole in a vessel is as the square of the height of water above the aperture.

1. A miller has a head of water 4 feet above the sluice; how high must the water be raised above the spring, so that half as much again water may be discharged from the sluice in the same time.

Thus, $\sqrt{4} = 2$, and half as much again is $2 + 1 = 3$, for the square of the required depth; therefore, $3 \times 3 = 9$ feet. Ans.

NOTE.—Water loses $\frac{2}{3}$ of its power in producing effects.

PENDULUM.

A pendulum is generally a rod of wire of metal, at the lower end of which a heavy piece or ball of metal is attached. The time of a vibration in a cycloid is the time of a heavy body's descent through half its length, as the circumference of a circle to its diameter; that is, as $3 \cdot 1416$ to 1: therefore (as a body descends freely by gravity through about $193 \cdot 5$ inches in the first second) to find the length of a pendulum vibrating seconds, use the following

RULE.—As $3 \cdot 1416^2 : 1 \times 1 :: 193 \cdot 5 : 19 \cdot 6$ inches, the half length; and $19 \cdot 6 \times 2 = 39 \cdot 2$ inches, the length.

To find the length of a pendulum that will vibrate any given time.

RULE.—Multiply the square of the seconds in any given time by $39 \cdot 2$, the product will be the length required.

1. Required the length of several pendulums, which will respectively vibrate $\frac{1}{4}$ seconds, $\frac{1}{2}$ seconds, seconds, minutes, and hours.

Thus, $\cdot 25^2 \times 39 \cdot 2 = 2 \cdot 45$ inches for $\frac{1}{4}$ seconds; $\cdot 5^2 \times 39 \cdot 2 = 9 \cdot 8$ inches for $\frac{1}{2}$ seconds; $1^2 \times 39 \cdot 2 = 39 \cdot 2$ inches for seconds,

as above; $60^2 \times 39{\cdot}2 =$ the inches in 2 miles, and 1200 feet for minutes; and 1 hour $= 3600$ seconds; therefore $3600^2 \times 39{\cdot}2$ $=$ the inches in 8018 miles, and 96 feet for hours.

2. What is the difference between the length of a pendulum which vibrates half seconds, and one which vibrates 3 seconds?

Thus, $\overline{3 \times 3 \times 39{\cdot}2} - {\cdot}5 \times {\cdot}5 \times 39{\cdot}2 = 343$ inches $= 27\frac{7}{12}$ feet. Ans.

To find the length of a pendulum for any number of vibrations in a minute at a particular place, the length of the pendulum which vibrates 60 *at the same place being known.*

RULE.—As the number of vibrations given is to 60, so is the square root of the length of the pendulum that vibrates seconds to the square root of the length of the pendulum required.

1. The length of a pendulum which vibrates seconds at New York is 39·1013 inches; what must be the length of one which must make 80 vibrations per minute?

Thus, $80 : 60 :: \sqrt{39{\cdot}1013}\ \frac{\sqrt{39{\cdot}1013} \times 60}{80} = 6{\cdot}253 \times 60$ $\div 80 = 375{\cdot}18 \div 80 = 4{\cdot}689$, the square root of the length of the pendulum required.

Then $4{\cdot}689 \times 4{\cdot}689 = 21{\cdot}9867$ inches. Ans.

2. What is the length of the pendulum which beats 70 times in a minute at London, where the seconds pendulum is 39·1393 inches in length? Ans. 28·751 inches.

The length of a pendulum being given, to find the number of vibrations in a minute.

RULE.—As the square root of the length of the pendulum given is to the square root of the length of that which beats seconds, so is 60 to the number of vibrations required.

1. How many vibrations will a pendulum 64 inches long make in a minute at New York, where the length of the pendulum vibrating seconds is 39·1013 inches?

Thus, $\sqrt{64} : \sqrt{39{\cdot}1013} :: 60 :$ number of vibrations.

$(\sqrt{39{\cdot}1013} \times 60) \div \sqrt{64} = (6{\cdot}253 \times 60) \div 8 = 375{\cdot}18 \div 8 = 46{\cdot}8975$ vibrations. Ans.

2. At Paris, the seconds pendulum being 39·1280, how many vibrations will a pendulum that is 62·5 inches long make in a minute? Ans. 47·476 vibrations.

3. How many vibrations will a pendulum 49 inches long make in a minute at Edinburgh, where that which vibrates seconds is 39·1555 inches? Ans. 53·631 vibrations.

Distances as determined by the Velocity of Sound.

It has been ascertained by experiments that sound passes through the air at the rate of 1142 feet per second, or through a mile in about $4\frac{2}{3}$ seconds; therefore any distance may be readily found in feet, by multiplying the time in seconds which the sound takes to reach the ear by 1142; or in miles, by multiplying the same by $\frac{3}{14}$.

The time between seeing a flash of lightning, or that of a gun, and hearing the report, may be observed by a watch or a seconds pendulum; or it may be determined by the beats of the pulse at about 70 to a minute for a person in health, or $5\frac{1}{2}$ pulsations for a mile. Sound of all kinds, it is ascertained, travels at the rate of 15 miles in a minute; the softest whisper travels as fast as the most tremendous thunder.

If a ship in distress fire a gun, the light of which is seen on shore, or by another vessel, 20 seconds before a report is heard, it is known to be at a distance of 1142×20, or a little more than $4\frac{1}{2}$ miles. If I see a vivid flash of lightning, and in 2 seconds hear a sharp clap of thunder, I know that the cloud is not more than 700 yards from the place, and a sufficient warning to retire from an exposed situation.

1. After observing a flash of lightning, it was 12 seconds before I heard the thunder; required the distance of the cloud from which it came.

$12 \times \frac{3}{14}$ $(12 \times 3 \div 14) = 36 \div 14 = 2\frac{4}{7}$ m. Ans.

2. How long, after the firing of a gun, may the report be heard, admitting the distance to be 8 miles in a straight line?

Thus, $8 \times 14 = 112 \div 3 = 37\frac{1}{3}$ seconds.

3. How far off was a cloud, from which thunder issued, whose report was 5 pulsations after the flash of lightning, counting 75 to a minute? Ans. $1522\frac{2}{3}$ yards.

Mechanical Powers.

There are six simple machines, which are called *Mechanical Powers;* they are the *Lever, Pully, Wheel* and *Axle, Inclined Plane, Wedge,* and the *Screw.*

Lever, or Steelyard.

The lever is a straight bar of wood or metal, which moves around a fixed point called the fulcrum. There are three kinds of levers: 1. When the fulcrum is between the weight and the power. 2. When the weight is between the power and the fulcrum. 3. When the power is between the fulcrum and the weight. The parts of the lever, from the fulcrum to the weight and power, are called the *arms* of the lever. An equilibrium is produced in all the levers when the weight multiplied by its distance from the fulcrum is equal to the product multiplied by its distance from the fulcrum; that is, *the weight is to the power, as the distance from the power to the fulcrum, to the distance from the weight to the fulcrum.*

1. In a lever of the first kind the fulcrum is placed at the middle point; what power will be necessary to balance a weight of 40 pounds?

2. A lever of the first kind is 8 feet long, and a weight of 60 pounds is at a distance of 2 feet from the fulcrum; what power will be necessary to balance it?

Ans. 20 pounds.

3. The weight upon the short arm of a lever is 100 pounds, and its distance from the fulcrum 8 inches; what power at the long arm, which measures 100 inches, would just balance the weight?

Thus, $100 \times 8 = 800 \div 100 = 8$ lbs. Ans.

4. If a man weighing 160 pounds, rest on the end of a lever 10 feet long, what weight will he balance on the other end, admitting the prop to be 1 foot from the weight?

The distance between the prop and power is $10 - 1 = 9$ feet; therefore as 1 foot : 9 ft. : : 160 lbs. : 1440 lbs. Ans.

5. If a weight of 1440 pounds is to be raised with a lever 10 feet long, and the prop fixed 1 foot from the weight, what power or weight applied to the other end of the lever would balance it?

As 9 : 1 : : 1440 : 160 lbs. Ans.

6. If a weight of 1440 pounds be placed 1 foot from the prop, at what distance from the prop must a power of 160 pounds be applied to balance it?

As 160 : 1440 : : 1 : 9 ft. Ans.

7. At what distance from a weight of 1440 pounds must a prop be placed, so that a power of 160 pounds applied 9 feet from the prop may balance it?

As 1440 : 160 : : 9 : 1 foot. Ans.

Wheel and Axle.

This machine is composed of a wheel and crank, firmly attached to a cylindrical axle. In order to balance the weight, we must have *the power to the weight as the radius of the axle to the length of the crank, or radius of the wheel.*

To find what forms will balance each other.

Rule.—The weight, multiplied by its distance from the fulcrum, which is the radius of the axle, is equal to the power multiplied by its distance from the same point, which is equal to the radius of the wheel.

1. Required the weight that can be held in equilibrium by a power of 28 pounds applied at the circumference of a wheel, whose diameter is 6 feet, the rope which supports the weight being wound round an axis whose radius is 8 inches.

The radius of the wheel is 3 ft. = 36 in.; and 28 × 36 = 1008 ÷ 8 = 126 lbs. the weight required.

2. The diameter of a wheel is 60 inches; required the diameter of the axle, so that 1 pound on the wheel may balance 10 pounds on the axle.

As 1 : 60 : : 10 : 6 inches, diameter required.

The Screw.

The power is to the weight which is to be raised, as the distance between two threads of the screw is to the circumference of a circle described by the power applied at the other end of the lever.

Rule.—Find the circumference of the circle described by the end of the lever; then as that circumference is to the distance between the spiral threads of the screw, so is the weight to be raised, to the power which will raise it.

1. In a screw, whose threads are 1 inch apart, the lever by which it is turned 30 inches long, and the weight to be raised one ton, or 2240 pounds, what power or force must be applied to the end of the lever sufficient to raise the weight?

Thus, the lever being the semi-diameter of the circle, the diameter is 60 inches; then 3·1416 × 60 = 188·496 inches, the circumference; therefore, as 188·496 in. : 1 in. : : 2240 lbs. : 11·88. Ans.

2. Let the lever be 30 inches, the circumference of which is

found to be 188·496, the threads 1 inch apart, and the power 11·88 pounds; required the weight to be raised.

As 1 : 188·496 : : 11·88 : 2240 lbs. nearly. Ans.

3. Let the weight be 2240 pounds, the power 11·88 pounds, and the lever 30 inches; required the distance between the threads.

As 2240 : 11·88 : : 288·496 : 1 nearly. Ans.

4. Let the power be 11·88 pounds, the weight 2240 pounds, and the threads 1 inch apart; required the length of the lever.

As 11·88 : 2240 : : 1 : 188·5 inches.

Then as 355 : 113 : : 188·5 : 60 in. *nearly*, the diameter; $60 \div 2 = 30$ inches. Ans.

5. What power is required to raise a weight of 8000 pounds, by a screw 12 inches in circumference, and 1 inch pitch?

Thus, 12 : 1 : : 8000 : $666\frac{8}{12}$ lbs. = the power at the circumference of the screw.

The Pulley.

The pulley is a wheel, having a groove cut in its circumference for the purpose of receiving a cord which passes over it. When motion is imparted to the cord, the pulley turns around its axis, which is generally supported by being attached to a beam above. Pulleys are divided into two kinds, fixed and movable pulleys. When the pulley is fixed, it does not increase the power which is applied to raise the weight, but merely changes the direction in which it acts.

One movable pulley gives one mechanical power; if several pulleys are used, the advantage gained is still greater, and a heavy weight may be raised by a small power, but more time will be required. There is also a loss of power by the resistance of the pulleys, called *friction*, which varies in different machines, and must be allowed for in calculating to do a given work. It is plain, that in the movable pulley, all the parts of the cord will be equally stretched; therefore each cord from one pulley to another will bear an equal part of the weight, consequently the *power will always be equal to the weight divided by the number of cords which reach from one pulley to another*, or,

Rule.—The weight is equal to the product of the power by twice the number of movable pulleys.

1. In a block and tackle, the movablé block contains five wheels; what power will be raised by it, with a weight of 380 pounds? $380 \times 10 = 3800$ lbs. Ans.

2. In two movable pulleys with 5 cords, what power will support a weight of 100 pounds? $100 \div 5 = 20$ lbs. Ans.

3. In a block and tackle, the movable block contains 8 wheels; what power will be raised by it, with a weight of 640 pounds?

The Inclined Plane.

The inclined plane is a slope or declivity, which is used for the purpose of raising weights. The quantity of weight is great in proportion to the inclination of the plane; and in consequence the difficulty of raising is greater, and the velocity, or relative quickness of motion less. The less the *inclination* of the plane the greater the pressure of the weight on the plane.

To find the weight that a given power will support on an inclined plane, the length of which is known.

Rule.—Multiply the power by the length of the plane, and divide the product by the height; the quotient is the weight that the power will support.

1. An inclined plane is 14 yards long, and 2 yards high; required the weight that will be supported on it by a power of 100 pounds.

Thus, $10 \times 14 \div 2 = 700$ pounds, the weight required.

2. The length of a plane is 30 feet, and its height 6 feet; what power will be necessary to balanee a weight of 200 lbs.? Ans. 40 pounds.

The height of a plane is 10 feet, and the length 20 feet; what weight will a power of 50 pounds support? Ans. 100 lbs.

The Wedge.

The wedge is composed of two inclined planes united together along their bases, and forming a solid. It is used to cleave masses of wood or stone. The wedge acts principally by being struck with a hammer or mallet on its head. As a theoretical rule, it may be said, that as the length of the wedge is to its back, so is the resistance to the power.

Miscellaneous Questions.

1. FROM a mahogany plank, 26 inches broad, a yard and a half is to be sawed off; what distance from the end must the line be struck? Ans. 6·23 feet.

2. A joist is $8\frac{1}{2}$ inches deep, $3\frac{1}{2}$ broad; what will be the dimensions of a scantling just twice as large as a joist that is 4·75 inches broad? Ans. 12·52 inches deep.

3. The two sides of an obtuse-angled triangle are 20 and 40 poles; what must the length of the third side be, so that the triangle may contain just one acre? Ans. 58·876.

4. What will the diameter of a globe be when the solidity and superficial content thereof are represented by the same number? Ans. 6.

5. The diameter of a Winchester bushel is 18·5 inches, and its depth 8 inches; what must the diameter of that bushel be whose depth is 7·5 inches?

Thus, $\sqrt{7 \cdot 5} : \sqrt{8} :: 18 \cdot 5 \; \dfrac{18 \cdot 5 \sqrt{8}}{\sqrt{7 \cdot 5}} = \dfrac{18 \cdot 5 \times 60}{7 \cdot 5} = 19 \cdot 1067$ inches. Ans.

6. How many 3 inch cubes can be cut out of a 12 inch cube? Ans. 64.

7. How many cubical inches in a block of marble $7\frac{1}{2}$ feet in length, $2\frac{1}{2}$ in width, and 9 inches in depth?

8. If a piece of land be 40 chains in length, and 35 in breadth, how many acres does it contain? Ans. 140.

9. How much in length that is 8 inches wide, will be equal to a square foot? Ans. 18 inches.

10. The diameter of a circle is 25 rods; required the length of a stone wall that will enclose it. Ans. 78·54 rods.

11. How many men may stand on an acre of land, allowing 12 square feet to each? Ans. 3630 men.

12. The diameter of a globe is 18 inches; what is its solidity? Ans. 1·76715 cubic feet.

13. What is the solidity of a cylinder whose length is 42 feet, and diameter 17 inches? Ans. 66·2+ cubic feet.

14. How many solid yards of earth must be removed to make a cellar 548 feet long, 27 feet wide, and 6 feet deep?

Ans. 288 solid yards.

15. How many pieces ¼ of an inch square are contained in a circular piece of plank 6 inches in diameter, and 1½ inches thick?

Ans. 2714+ pieces.

16. The area of an equilateral triangle is 24 acres; what is the length of its sides? Ans. 23·5426 chains.

17. How many feet in a pile of wood 18 feet in length, 4 feet in width, and 7·5 feet in depth?

18. If the area of a circle be 184·125, what is the side of a square equal in area? $\sqrt{184{\cdot}125} = 13{\cdot}569 +$ Ans.

19. If the area of a triangle be 160, what is the side of a square equal in area? $\sqrt{160} = 12{\cdot}649 +$. Ans.

20. If it be required to place 2016 men so that there may be 56 in rank, and 36 in file, and to stand 4 feet distance in rank and file, how much ground will they stand on?

NOTE.—To solve the above question, or any of them, use the following proportion:

As 1, or unity (:) is to the distance (: :) so is the number in rank less by one (:) to a fourth number.

Next, do the same by the file, and multiply the two numbers together found by the above proportion, and the product will be the answer.

Thus, as $1 : 4 :: 56 - 1 : 220$; and as $1 : 4 :: 36 - 1 : 140$; then $220 \times 140 = 30800$ square feet. Ans.

The above rule will be useful in planting trees, having the distance between each given.

21. It is required to set out an orchard of 600 trees, so that the length shall be to the breadth, as 3 to 2; and the distance of each tree from the other 7 yards, or 21 feet; how many trees must it be in length, and how many in breadth; and how many square yards of ground do they stand on?

RULE.—To solve any question similar to the preceding, say as the ratio in length (:) is to the ratio in breadth (: :) so is the number of trees (:) to a fourth number, whose square root is the number in breadth. And as the ratio in breadth (:) is to the ratio in length (: :) so is the number of trees to a fourth whose square root is the number in length.

Thus, as $3 : 2 :: 600 : 400$, and $\sqrt{400} = 20 =$ number in breadth.

As $2 : 3 :: 600 : 900$; and $\sqrt{900} = 30 =$ number in length.

Then, as $1 : 7 :: 30 - 1 : 203$, and as $1 : 7 :: 20 - 1 : 133$.

And $203 \times 133 = 26999$ square yards. Ans. ($20 \times 30 = 600$ trees.)

22. If a bullet 6 inches in diameter weigh 32 pounds, what will a bullet of the same metal weigh whose diameter is 3 inches?

$6^3 = 216 : 3^3 = 27$: as $216 : 32 :: 27 : 4$ lbs. Ans.

23. If a globe of silver of 3 inches diameter be worth \$45, what is the value of another globe of a foot diameter?

Ans. 2880 dollars.

24. There is an island 50 miles in circumference, and 3 men start together to travel the same way around it; A goes 7 miles per day; B 8; and C 9; when will they all come together again; and how far will each travel?

Ans. $\overline{50 \times 7} + \overline{50 \times 8} + \overline{50 \times 9} \div \overline{7 + 8 + 9} = 50$ days.

Ans. A 350; B 400; C 450 miles; all will meet.

25. A line 35 yards long, will exactly reach from the top of a fort, standing on the brink of a river, known to be 27 yards broad to the opposite bank; required the height of the wall.

Ans. 22 yards, $7\frac{3}{4}$ inches, *nearly*.

26. Being about to plant 5229 trees equally distant in rows, the length of the grove is to be three times the breadth, how many of the shorter rows will there be?

One-third of the trees are to form an exact square; the side whereof being 42, shows how many come into a short row.

27. Required the length of a shore, the bottom of which being set 9 feet from the perpendicular side of a house, will support a weak place in the wall $22\frac{1}{2}$ feet from the ground.

Ans. 24 ft. $2\frac{3}{4}$ in.

28. Light passes from the sun to the earth in 8·2 minutes; in what time would it pass from the sun to Herschel, it being 1803930416·66 miles?

As the earth's distance from the sun is $94772980 : 8{\cdot}2 :: 1803930416{\cdot}66 : 2$ h. 36 m. 4″ 50‴. Ans.

29. Allowing the density of the moon to be 464, and that of the earth 392·5, required the proportion between the quantity in the earth and in that of the moon, allowing the earth's diameter to be 7964·12, and that of the moon 2180 miles, and supposing the earth a perfect sphere, (which however it is not.)

$$\frac{7964{\cdot}12 \times 7964{\cdot}12 \times 7964{\cdot}12 \times 392{\cdot}5}{2180 \times 2180 \times 2180 \times 464} = 41{\cdot}24 \text{ times the}$$

quantity of matter in the earth that there is in the moon; or the weight of the earth is so many times that of the moon.

30. The mean diameter of the earth's orbit, (or annual path

round the sun) supposing it a circle, is in miles 190437141·7; required its mean motion (or the space which it moves through in its orbit) per minute.

Thus, $190437141{\cdot}7 \times 3{\cdot}1416 = 598277324{\cdot}36$ miles in circumference.

Then as days, $365{\cdot}25 : 598277324{\cdot}36 :: 1' : 1137{\cdot}49$ miles. Answer.

The decimal motion of the earth on its axis, is 17·25 miles per minute at the equator.

31. Required the axis of a globe whose solidity may be just equal to the area of its surface.

$$\cdot7854 \times 4 \div \cdot5236 = 6 \text{ inches. Ans.}$$

32. I have a square stick of timber 18 inches by 14, but one containing one-third part of the timber will answer the purpose, how wide will it be?

$$\frac{18 \times 14}{3} \div 8 = 10\tfrac{1}{2} \text{ inches. Ans.}$$

33. A lent B a solid stack of hay, measuring 20 feet every way; subsequently B returned a quantity, measuring 10 feet; what proportion of the hay remains due?

$$20^3 - 10^3 = 7000 \text{ feet} = \tfrac{7}{8}. \text{ Ans.}$$

34. A ship's hold is 75·5 feet long, 18·5 wide, and 7¼ deep; how many bales of goods 3·5 feet long, 2·25 deep, and 2·75 wide, may be placed therein, leaving a gangway the whole length of 3·25 feet wide?

$$\text{Thus, } \frac{\overline{75{\cdot}5 \times 18{\cdot}5 \times 7{\cdot}25 - 75{\cdot}5 \times 7{\cdot}25 \times 3{\cdot}25}}{3{\cdot}5 \times 2{\cdot}25 \times 2{\cdot}75} = 385{\cdot}44$$

bales. Ans.

35. I would have a cubic box made, capable of securing just 50 bushels, the bushel containing 2150·425 solid inches; what will be the length of the side?

$$\sqrt[3]{2150{\cdot}425 \times 50} = 47{\cdot}55 \text{ inches. Ans.}$$

36. If a circular pillar 9 inches in diameter contain 5 feet, required the diameter of a column of equal length, which measures 10 times as much.

Thus, $5 : 9 \times 9 :: 5 \times 10 : 810$; and $\sqrt{810} = 28{\cdot}46$ inches. Answer.

37. In a pile of stone 20 feet in length, 8·5 feet in width, and 4·25 feet in height, how many perches?

Ans. 29·191+ perches.

38. Required the number of bushels of charcoal in a box 15·5 feet in length, 3·5 feet in width, and 3 feet in height.

Ans. 100 bush. 4 qts.

39. Required the number of cubical feet in a stick of 30 feet in length, 1·5 feet in width, and ½ foot in depth.

Ans. 22·5 feet.

40. A cellar contains 1728 cubical feet, its length is 16 feet, and breadth 12 feet; required the depth. Ans. 9 feet.

41. Required the number of bushels of potatoes in a bin 6 feet in length, 4·5 feet in width, and 5·75 feet in depth.

Ans. 105·0555 = 105 bushels 1 pint.

42. If a vessel contains 3600 cubical feet of wood upon her deck, and is 30 feet in length and 20 feet in width, required the height of the wood. Ans. 6 feet.

43. In 221184 solid inches, how many cords?

44. How many cubical inches, and cubical feet, in a stick of timber 6 inches square and 4 feet in length?

45. Required the number of cubical yards in digging a canal 1 mile in length, 20 feet in width, and 6 feet in depth; and what the digging will cost at 6 cents per yard.

Ans. 23466⅔ yards; cost, $1408.

46. A man paved a court 157 feet by 12 feet, at 3 cents per square yard; how many square yards did he pave, and how much did he receive? Ans. 209⅓ sq. yds.; amount, $6·25.

47. How many solid and square feet are contained in a stick of timber 70 feet in length, 15 inches in depth, and 18 inches in width? Ans. 131·25 cubic feet = 1575 square feet.

48. Required the content of a plank 30 feet in length, 22 inches in width, and 2 inches in thickness. Ans. 110 ft.

49. Required the quantity and value of a load of charcoal 16 feet in length, 3½ feet in width, and 4 feet in height, at 10 cents per bushel. Ans. 151·57 + $15·157 value.

50. How many feet in a board 18 feet in length, and 16 inches in width? Ans. 24 feet.

51. Required the number of books that are 7 inches in length, 4·5 inches in width, and 5 inches in thickness, to fill a trunk that measures 35 inches in length, 18 inches wide, and 15 inches deep. Ans. 600.

52. How many tons of timber in two sticks, each 30 feet long, 20 inches wide, and 12 inches thick?

53. How much will a marble slab cost that is 7 feet 4 inches long, 1 foot 3 inches wide, at $1 per foot?

Ans. 9·16⅔.

54. The cubic inch of common glass weighs about 1·36 ounces troy; salt water ·5427 ounces; brandy ·48927 ounces. A seaman has a gallon of brandy in a bottle, which weighs 4½ pounds troy, out of water, and to conceal it, throws it overboard into salt

water; will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water?

Thus, $4\frac{1}{2}$ pounds = 54 ounces = weight of bottle $\frac{54}{1{\cdot}36}$ =

$$\begin{array}{ll} 39{\cdot}7059 & \text{cubic inches in bottle.} \\ \underline{231\phantom{{\cdot}7059}} & \text{in brandy.} \\ 270{\cdot}7059 & = \text{in both.} \end{array}$$

Then $270{\cdot}7059 \times {\cdot}5427 = 146{\cdot}912$ oz. = weight of salt water occupied by the bottle and brandy.

And ·48927 (= weight of a cubic inch of brandy) $\times\ 231 = 113{\cdot}02 +$ oz.; and $113{\cdot}02 + 54 = 167{\cdot}02$ oz. = weight of the bottle and brandy.

From this, take the weight of salt water, 146·192 oz. Ans.

Supposing the bottle full, it is 20·11 ounces heavier than the same bulk of salt water, and consequently will sink.

55. Purchased a box of window glass, containing 256 panes of glass 8 by 10 inches; required the number of square feet, and the cost at 5 cents per foot.

$10 \times 8 = 80$ in.; $256 \times 80 = 20480 \div 144 = 142{\cdot}22$ square feet; $7·1110. Ans.

56. Bought a box of window glass containing 320 square feet of glass 10 by 12; how many panes? Ans. 384.

57. Required the length of a stone wall that will enclose a circle whose diameter is 25 rods. Ans. 78·54 rods.

58. If a piece of land be 40 chains in length, and 35 in breadth, how many acres does it contain? Ans. 140.

59. How many bushels of grain will a cubical box contain whose side is $4\frac{1}{2}$ feet? Ans. 73·2247 bushels.

Contents.